florentín smarandache

nidus idearum
Viva la Neutrosophia!

neutrosophy

spheroid neutrosophic set

neutrosophic set

neutrosophic psychology

symbolic neutrosophic theory

neutrosophic cubic set

neutrosophic duplets & triplets

neutrosophic statistics

neutrosophic numbers

neutrosophic physics

neutrosophic logic

neutrosophic algebraic structures

neutrosophic precalculus and calculus

law of included multiple-middle

unparticle

unmatter

pons
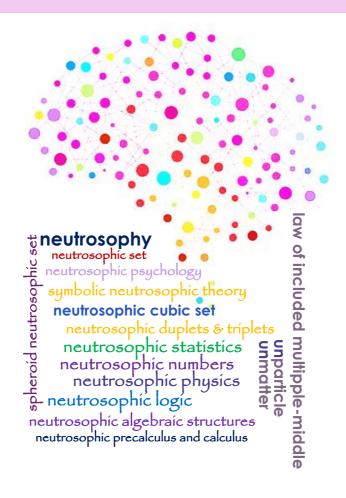
Florentin Smarandache

Nidus idearum.
Scilogs, III: Viva la Neutrosophia!

Brussels, 2017

Exchanging ideas with A. A. A. Agboola, Swati Aggarwal, Mumtaz Ali, Young Bae Jun, E. Barrenechea, Kanika Bhutani, Sisalah Bouzina, Said Broumi, Cuong Bui Cong, H. Bustince, Emenia Cera, Victor Christianto, Luu Dat, Jean Dezert, Huda Esmail, C. Franco, Gaurav Garg, Hewayda ElGhawalby, Ervin Goldfain, D. Gómez, Muhammad Gulistan, S. A. El-Hafeez, Omar Hammoui, Nasruddin Hassan, Pham Hong Phong, Martina Jency, Akira Kanda, J. Fernandez, Ilanthenral Kandasamy, W. B. Vasantha Kandasamy, Qaisar Khan, Volodymyr Krasnoholovets, Megha Kumar, Santanu Kumar Patro, Paul Kwan, Francisco Gallego Lupiañez, Chunfang Liu, P. Majumdar, Arnaud Martin, Nguyen Minh, Kalyan Mondal, Javier Montero, E. M. El-Nakeeb, M. Pagola, David Paul, Surapati Pramanik, Ashraf Al-Quran, Dmitri Rabounski, Nouran M. Radwan, Akbar Rezaeii, Alaa El Din M. Riad, Arsham Borumand Saeid, A. A. Salama, U. Samanta, Gheorghe Săvoiu, M. Badr Senousy, Prem Kumar Singh, Le Huang Son, Ridvan Şahin, Mehmet Şerhat Can, Zenonas Turskis, Jun Ye, Fu Yuhua, Edmundas Kazimieras Zavadskas, Hong-yu Zhang, Sarfaraz Hashemkhani Zolfani

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FOREWORD

Welcome into my scientific lab!

My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific meditation and chats: *a nest of ideas* (**nidus idearum**, in Latin). I called the jottings herein *scilogs* (truncations of the words *scientific*, and gr. Λόγος – appealing rather to its original meanings "ground", "opinion", "expectation"), combining the welly of both science and informal (via internet) talks (in English, French, and Romanian).

<div align="center">*</div>

In this *third book of scilogs* collected from my nest of ideas, one may find new and old questions and solutions, referring to topics on NEUTROSOPHY – email messages to research colleagues, or replies, notes about authors, articles, or books, so on. Feel free to budge in or just use the *scilogs* as open source for your own ideas!

<div align="center">*</div>

Neutrosophy is a new branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

I coined the words "neutrosophy" and "neutrosophic" in my 1998 book: Florentin Smarandache, *Neutrosophy. Neutrosophic Probability, Set, and Logic*, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998; http://fs.gallup.unm.edu/eBook-neutrosophics6.pdf (last edition online). Etymologically, "neutro-sophy" (noun)

<div align="center">5</div>

[French neutre < Latin neuter, neutral, and Greek sophia, skill/wisdom] means knowledge of neutral thought, while "neutrosophic" (adjective), means having the nature of, or having the characteristic of Neutrosophy.

*

The most important evolutions in neutrosophics, in chronological order:

- 1998 - introduction of neutrosophic set / logic / probability / statistics
- 2005 - introduction of interval neutrosophic set/logic
- 2013 - development of neutrosophic probability (chance that an event occurs, indeterminate chance of occurrence, chance that the event does not occur)
- 2013 - refinement of components (T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...)
- 2014 - development of neutrosophic statistics (when indeterminacy is introduced to the classical statistics)
- 2015 - introduction of neutrosophic precalculus and neutrosophic calculus
- 2015 - introduction of neutrosophic dynamic systems, symbolic neutrosophic logic, etc.

F.S.

Links to some books on Neutrosophy:

- http://fs.gallup.unm.edu/eBook-Neutrosophics5.pdf
- http://fs.gallup.unm.edu/INSL.pdf
- http://fs.gallup.unm.edu/NeutrosophicMeasureIntegralProbability.pdf
- http://fs.gallup.unm.edu/NeutrosophicStatistics.pdf
- http://fs.gallup.unm.edu/NeutrosophicPrecalculusCalculus.pdf
- http://fs.gallup.unm.edu/SymbolicNeutrosophicTheory.pdf

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TOPICS

1 Neutrosophic Set

Compared with all other types of sets, in the neutrosophic set each element has three components which are subsets (not numbers as in fuzzy set) and considers a subset, similarly to intuitionistic fuzzy set, of "indeterminacy" - due to unexpected parameters hidden in some sets, and let the superior limits of the components to even boil *over 1* (overflooded) and the inferior limits of the components to even freeze *under 0* (underdried).

For example: an element in some tautological sets may have t > 1, called "overincluded". Similarly, an element in a set may be "overindeterminate" (for i > 1, in some paradoxist sets), "overexcluded" (for f > 1, in some unconditionally false appurtenances); or "undertrue" (for t < 0, in some unconditionally false appurtenances), "underindeterminate" (for i < 0, in some unconditionally true or false appurtenances), "underfalse" (for f < 0, in some unconditionally true appurtenances).

This is because we should make a distinction between unconditionally true (t > 1, and f < 0 or i < 0) and conditionally true appurtenances (t ≤ 1, and f ≤ 1 or i ≤ 1).

2 Neutrosophic Statistics

To Prof. Paul Kwan, Prof. David Paul, Mumtaz Ali:

About using the statistical analysis in research in agriculture and other fields, may I attach for you a book on Neutrosophic Statistics, that deals with indeterminacy

inserted into statistics (i.e. sample or population size not well-known, individuals not 100% belonging to the sample or population (I mean some individuals only partially belong to the sample or population, which happens in our everyday life) etc. We can discuss about applying neutrosophic statistical analysis in modeling the drought forecasting and others.

3 Neutrosophic Probability

To Prof. Paul Kwan, Prof. David Paul, Mumtaz Ali:

By the way, I also generalized the classical probability to a neutrosophic probability (*NP*), in the following way:

> *NP*(event) = (chance that the event occurs, indeterminate-chance, chance that the event does not occur), so there are three components.

For example, the proposition: *P* = "It will be raining next week", has a chance of occurring, a chance of not occurring, and an indeterminate (vague, confused, unknown) chance.

We can use the neutrosophic probability (because it includes the indeterminacy, which is very common in our everyday life) in weather prediction for agriculture, in cancer prediction etc.

Mumtaz Ali:

In the last few years, a significant research papers have been published around the world to model the

environmental parameters such as temperature, rain fall, humidity etc. to predict their impacts of agriculture. My current supervisor (Dr. Ravi) has also gone to China in this regard a few days ago.

He is an expert in the area of Environmental and Agriculture Engineering. He is working to apply *ML* (such as neural networks, *SVM*, *ELM*, statistical analysis) in this area.

I will be the first one in his group to apply fuzzy logic, neutrosophic logic and statistical models in the field.

4 Neutrosophic Psychology

To Ilanthenral Kandasamy:

Neutrosophic Psychology means indeterminacy studied in psychology, and connection of opposite theories and their neutral theories together.

If a scale weights are, for example, 1, 2, 3, 4, 5, 6, 7, we can refine in many way, for example:

- pessimistically as T, I_1, I_2, I_3, I_4, I_5, F;
- or optimistically as T_1, T_2, I_1, I_2, I_3, F_1, F_2;
- or more optimistically T_1, T_2, T_3, I, F_1, F_2, F_3; etc.

Many ideas can be developed on the REFINED neutrosophic set.

5 Neutrosophic Duplets & Triplets

To Mumtaz Ali:

We can operate directly on neutrosophic duplets $<a, \text{neut}(a)>$, and on neutrosophic triplets $<a, \text{neut}(a), \text{anti}(a)>$.

6 Positively or Negatively Qualitative Neutrosophic Components

To Muhammad Gulistan:

Here it is the general picture on the neutrosophic components *T, I, F*:

- the *T* is considered a positively (good) qualitative component;
- while *I* and *F* are considered the opposite, i.e. negatively (bad) qualitative components.

When we apply neutrosophic operators, for *T*'s we apply one type, while for *I* and *F* we apply an opposite type.

Let's see examples:

- neutrosophic conjunction:

$<t_1, i_1, f_1> \wedge <t_2, i_2, f_2> = <t_1 \wedge t_2, i_1 \vee i_2, f_1 \vee f_2>$, as you see we have t-norm for t_1 and t_2, but t-conorm for i_1 and i_2, as well as for f_1 and f_2;

- neutrosophic disjunction:

$<t_1, i_1, f_1> \vee <t_2, i_2, f_2> = <t_1 \vee t_2, i_1 \wedge i_2, f_1 \wedge f_2>$ Etc.

Now, about Definition 1 in the paper *Neutrosophic Cubic Ideals* the characteristic neutrosophic set function:

- for μ we need to have: 1 if x $\in$ G, 0 otherwise; and the opposite for the next two components I and F: 0 if x $\in$ G, 1 otherwise; 0 if x $\in$ G, 1 otherwise.

For λ we have: 0 if x $\in$ G, 1 otherwise; and the opposite for I and F: 1 if x $\in$ G, 0 otherwise; respectively 1 if x $\in$ G, 0 otherwise.

Is this answer satisfactory for you?

There also are authors who prefer that *T* and *I* are computed in the same way, while *F* is computed differently.

There are such interpretations for the neutrosophic disjunction, depending on the experts:

Pessimistic Interpretation:

- for μ you should use: $\vee$, $\wedge$, $\wedge$;

- for λ you should use $\wedge$, $\vee$, $\vee$.

Optimistic Interpretation:

- for μ you should use: $\vee$, $\vee$, $\wedge$;

- for λ you should use $\wedge$, $\wedge$, $\vee$.

7 Imaginary Indeterminacy

To Santanu Kumar Patro:

About your "imaginary indeterminacy" which is indeterminacy in sub-conscience or indeterminacy in sleeping time, I agree with it. Try to get more such concrete examples and use them within the frame of the neutrosophic set.

8 Neutrosophic Implication

In neutrosophic logic we work with partial truth, more exactly a proposition has a (*t, i, f*) logical value, where *t* is the degree of truth, *i* the degree of indeterminacy and *f* the degree of falsehood.

Similarly, when we talk about neutrosophic implication: P → Q, where P and Q are neutrosophic propositions, P(t_P, i_P, f_P) and Q(t_Q, i_Q, f_Q) respectively, then we have an implication which is partially time: (P→Q)$_{(tP→Q, iP→Q, fP→Q)}$.

9 Neutrosophic Cubic Set

To Young Bae Jun (founder of cubic set in 2012):

We can extend even further the neutrosophic cubic set, by considered the *n*-valued refined neutrosophic set, i.e. *T* is refined in types of truths: *T₁, T₂, ...*; similarly *I* is refined in types of sub-indeterminacies *I₁, I₂, ...*; and *F* is refined in types of falsehoods *F₁, F₂, ...* . So, we can get a *refined neutrosophic cubic set*.

Similarly as in neutrosophics, one can refine the fuzzy set, and we can make a *refined fuzzy cubic set*.

10 Transdisciplinarity

To Mumtaz Ali:

In transdisciplinarity there is overlapping between disciplines, contradictory information in the connected disciplines, and the boundaries between some disciplines

are not clear. That's why we can use the neutrosophic set and logic, which deal with indeterminacy.

11 Generalization of Neutrosophic Vague Soft Expert Set

To Ashraf Al-Quran and Nasruddin Hassan:

We can generalize the *Neutrosophic Vague Soft Expert Set*, from a second neutrosophic point of view, i.e. if we consider the opinion set $O_1 = \{1 = agree, 0 = disagree\}$ extended to $O_2 = \{1 = agree, 1/2 = indeterminate\ agreement / disagreement, 0 = disagree\}$, or we may denote $O_2 = \{1 = agree, I = indeterminate\ agreement/disagreement, 0 = disagree\}$ by using the symbol "*I*" for indeterminacy as in I-Neutrosophic Algebraic Structures.

And also, then we may generalize further the above to the *Refined Neutrosophic Vague Soft Expert Set*.

12 Planets' Orbits from a Neutrosophic Point of View

In the book on *Neutrosophic Statistics*, and look inside for neutrosophic graphs (which are thick curves, not thin curves), and also neutrosophic diagrams (that have indedetrminacy into the diagrams).

In another book, about *Neutrosophic Probability*, one has for each event *E*:

(chance(E), indeterminatechance(E), nonchance(E)).

For example, if you roll a die on an irregular surface (surface with cracks) is possible that the die falls with a

vertex or an edge into the crack, so you can not exactly read the die (i.e. indeterminacy) - see the front cover of this book.

Neutrosophic statistics and neutrosophic probability better characterize the planets elliptical varying orbits, eccentricity, perihelion (closest distance to the Sun), and aphelion (farthest distance to the Sun).

13 Special Dual Like Numbers

The *special dual like numbers* (of the form *a+bh*, where *a* and *b* are real, while $h^2 = h$) are *exact* real values, while the neutrosophic numbers (of the form *a+bI*, where *a* and *b* are real, while *I* = indeterminacy, with $I^2 = I$) are *indeterminate* real values.

14 Subindeterminacies of the form $I_a = a/0$

In the book "Natural Neutrosophic Numbers and MOD Neutrosophic Numbers" (2015), the authors W. B. Vasantha Kandasamy, Ilanthenral K., F. Smarandache, denoted *a/0 = I (indeterminacy),* for any real number *a,* in a neutrosophic interpretation.

We now extend it to refined indeterminacies. We denote this as: $I_a = a/0$.

For example:

$0/0 = I_0$, $0/1 = I_1$, $0/2 = I_2$, or three subindeterminacies.

Then, $I_0/I_0 = I_0$,

because $(0/0)/(0/0) = (0/0)\cdot(0/0) = (0\cdot0)/(0\cdot0) = 0/0 = I_0$.

$I_1/I_1 = (1/0)/(1/0) = (1/0)\cdot(0/1) = 0/0 = I_0$.

Similarly $I_2/I_2 = (2/0)/(2/0) = (2/0)\cdot(0/2) = 0/0 = I_0$.

$I_1/I_2 = I_2/I_1 = 0/0 = I_0$.

For the multiplication, the same:

$I_1 \cdot I_2 = (1/0) \cdot (2/0) = 2/0 = I_2$, etc.

We can also do, in general,

- additions:

$I_a + I_b = a/0 + b/0 = (a+b)/0 = I_{(a+b)(mod\ n)}$

- and subtractions:

$I_a - I_b = a/0 - b/0 = (a-b)/0 = I_{(a-b)(mod\ n)}$

- and scalar multiplications:

$c(I_a) = c(a/0) = (ca)/0 = I_{(ca)(mod\ n)}$

- and powers:

$(I_a)^k = (a/0)^k = (a^k)/(0^k) = (a^k)/0 = I_{(a^k)(mod\ n)}$.

15 Symbolic Neutrosophic Theory

To Prof. Jun Ye:

I cited you several times in my last book ("Symbolic Neutrosophic Theory", 2015).

I re-send it to you, since many things may inspire you for doing more papers and research. For example, see various types of neutrosophic numbers, then neutrosophic dynamic systems (almost all systems are neutrosophic, since they are not perfectly isolated from the environment and because they have indeterminacies), we have also *(t, i, f)*-structures (meaning structures on sets whose elements do not completely belong to the set, but only partially), then refined indeterminacy *{I₁, I₂, ...}* and refined *T* and refined F, etc.

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If you have any questions, always write to me please. It is a great pleasure for me to exchange ideas with a talented scientist like you.

{By the way, I was four times in China having presentations on neutrosophics, in many cities.}

16 Connexions neutrosophiques

Omar Hammoui, etudiant doctorant :

Je suis le doctorant en philosophie en Algérie qui travaille sur la neutrosophie, j'ai besoins que vous m' oriente si c'est possible sur une documentation concernant le troisième chapitre de ma thèse qui porte sur les trois théories philosophique : théorie de la connaissance neutrosophique (*neutrosophic knowledge theory*), la théorie ontologique neutrosophique (*neutrosophic onto-logic theory*) simplement la théorie du reel neutro-sophique ; et en fin la théorie axiologique neutrosophique (*la morale et l'art neutrosophique*).

Vers Omar Hammoui :

Neutrosophie s'intéresse aux trois parties. Soit <A> une idée, une entité, une théorie, une notion, etc. Alors <antiA> est l'oppose de <A>, et <neutA> et la neutralité entre <A> et <antiA>.

La neutrosophie étudie la connexion et même le mélange entre les trois : <A>, <neutA>, et <antiA>.

La neutrosophie est une généralisation de la dialectique de Hegel, car la dialectique s'occupe seulement

de la connexion entre <A> et <antiA>, mais elle ignore la neutralité <neutA> qui a une grande importance, car la neutralité influence les deux opposites <A> et <antiA>.

17 *Fusion et Neutrosophie*

Au Prof. Arnaud Martin:

Il faut regarder le problème d'un autre angle, pas exactement celui de la fusion (qui este devenu routine). On parle surtout des 'probabilités subjectives' données par deux sources, en lieu des 'masses'. Donc, une source / expert / observateur donne une probabilité (estimation) : chance que l'avion soit ami, chance qu'il soit neutre, et chance qu'il soit ennemi. Une autre source donne une autre probabilité.

Ensuite, on combine les deux probabilités affin d'obtenir la meilleure probabilité subjective.

J'ai défini la probabilité neutrosophique, avec des exemples. Vas directement au chapitre de *Neutrosophic Probability*.

Le problème me parait bien pose, je l'ai dit :
Suppose an airplane A is detected by the radar.
Is it a friend, enemy, or neutral?
L'espace (le cadre) des probabilités subjectives est :
{F, N, E}, ou F = *Friend*, N = *Neutral*, E = *Enemy*.

On combine des probabilités subjectives, différentes des masses. C'est une nouvelle façon d'aborder les choses dans la fusion. Il faut que tu sois ouvert au nouveau, pas enferme dans la routine des méthodes de la fusion.

18 Neutrosophic Probability of an Event

In probability: there is an event A = {an airplane detected is friendly}.

What is the neutrosophic probability of A?

We say, $NP(A)$ = ?

Neutrosophic Probability of the event A = ?

$NP(A)$ = [chance(A is true), indeterminate-chance(A), chance(A is false)]

or

$NP(A)$ = [chance(*friend*), chance(*neutral*), chance(*enemy*)].

I inferred / combined two subjective opinions / probabilities of two sources. Here it is, similarly to combining two masses.

19 Neutrosophic Triplet Group

To Dr. Adesina Agboola & Mumtaz Ali:

The distinctions between Molaei's Generalized Group (MGG) and Neutrosophic Triplet Group (NTG) are:

- in MGG for each element there exists a unique neutral element, which can be the group neutral element; while in NTG each element may have many neutral elements, and also the neutral elements have to be different from the unique group neutral element;
- in MGG the associativity applies, in NTG the associativity is not required;

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- in MGG there exists a unique inverse of an element, while in NTG there may be many inverses for the same given element;
- MGG has a weaker structure than NTG.

20 Various Neutrosophic Sets

A cubic set C in X is a structure of the form:
$$C = \{(x, A(x), \lambda(x)), x \in X\},$$
where $A(x)$ is an interval-valued fuzzy set in X, and λ is a fuzzy set in X.

What about instead of "interval-valued fuzzy set" we take a "sub-unitary-valued fuzzy set", where the sub-unitary set is an extension of the interval.

Therefore:
$$EC = \{(x, B(x), \lambda(x)), x \in X\},$$
where $B(x)$ is a subunitary-valued fuzzy set in X, and λ is a fuzzy set in X.

Even more complex, we can say:
$$EEC = \{(x, B(x), A(x), \lambda(x)), x \in X\},$$
where $B(x)$ is a subunitary-valued fuzzy set in X, $A(x)$ is an interval-valued fuzzy set in X, and λ is a fuzzy set in X. Then we can study the cases when subunitary-valued fuzzy set $B(x)$ includes the interval-valued fuzzy set $A(x)$, and/or $A(x)$ includes the single-valued fuzzy set λ.

Similarly for the neutrosophic cubic set NC in X:
$$NC = \{(x, (A_{T(x)}, A_{I(x)}, A_{F(x)}), (\lambda_{T(x)}, \lambda_{I(x)}, \lambda_{F(x)}), \\ x \in X\},$$

where $(A_{T(x)}, A_{I(x)}, A_{F(x)})$ is an interval-valued neutrosophic set in X, and $(\lambda_{T(x)}), \lambda_{I(x)}, \lambda_{F(x)}$)is a single-valued neutrosophic set in X.

Then we extend it to:

ENC = $\{(x, (B_{T(x)}, B_{I(x)}, B_{F(x)}), (\lambda_{T(x)}), \lambda_{I(x)}, \lambda_{F(x)}), x \in X\}$,

where $(B_{T(x)}, B_{I(x)}, B_{F(x)})$ is a subunitary-valued neutrosophic set in X, and $(\lambda_{T(x)}), \lambda_{I(x)}, \lambda_{F(x)}$) is a single-valued neutrosophic set in X.

Even more complex:

EENC = $\{(x, (B_{T(x)}, B_{I(x)}, B_{F(x)}), (A_{T(x)}, A_{I(x)}, A_{F(x)}),$
$(\lambda_{T(x)}), \lambda_{I(x)}, \lambda_{F(x)}), x \in X\}$,

where $(B_{T(x)}, B_{I(x)}, B_{F(x)})$ is a subunitary-valued neutrosophic set in X, $(A_{T(x)}, A_{I(x)}, A_{F(x)})$ is an interval-valued neutrosophic set in X, and $(\lambda_{T(x)}), \lambda_{I(x)}, \lambda_{F(x)}$) is a single-valued neutrosophic set in X.

And again, to consider the situations when the neutrosophic subunitary-valued set B(x) includes the neutrosophic interval-valued set A(x), and/or A(x) includes the single-valued neutrosophic set λ.

21 Hypercomplex Rough Neutrosophic Numbers

To Kalyan Mondal & Surapati Pramanik:

After reading your interesting paper called "Tri-complex Rough Neutrosophic Similarity Measure and its Application in Multi-attribute Decision Making", I think you can generalize it to "Hypercomplex Rough Neutrosophic..." in a new paper, where the tri-complex

number is extended to the n-complex (also named hypercomplex).

I also defined an easier multiplication of the n-complex $h_j h_k = h_{j+k \ (mod \ n)}$ units (using the modulo *n* approach), which is easier than that defined by S. Olariu.

22 Neutrosophic Vague Set

To Akira Kanda:

I have introduced a general definition of membership, i.e. a subset of [0, 1], not necessarily a number or an interval of [0, 1] as in vague set.

Also, we have combined the neutrosophic set with the vague set, and got a neutrosophic vague set.

Neutrosophic set is as generalization of intuitionistic fuzzy set.

Vague set is also a part of intuitionistic fuzzy set.

*

We can consider graphs of other forms:

- the vertex is *I* (indeterminate);

- while the edge is (*t, i, f*).

Or the vertex = (*t, i, f*) and the edge is *I* (indeterminate).

Or both the vertex and the edge are *I* (indeterminate).

*

Example of first type:

```
A <- - - - - (0.4, 0.5, 0.7) - - - - -> B
∧
.
```

. {indeterminate edge between vertexes A and C}
.
.
∨
C

where vertex A = indeterminate (unknown), vertex B = hard work, vertex C = high performance.

So, at least a vertex is not known, i.e. one not knows what it represents.

23 Single-Valued Neutrosophic Graphs

To Said Broumi:

I saw that that in fuzzy graphs and intuitionistic fuzzy graphs there is the restriction that $T(u, v) \leq \min\{T(u), T(v)\}$, and respectively $F(u, v) \geq \max\{F(u), F(v)\}$.

For the neutrosophic fuzzy graphs also

$T(u,v) \leq \min\{T(u), T(v)\}$

and $I(u,v) \geq \max\{I(u), I(v)\}$, $F(u,v) \geq \max\{F(u), F(v)\}$.

I think we do not need any restriction for the neutrosophic graphs, since if the vertices u and v belong to the graph each of them in a certain degree of membership, that does not mean that the edge uv (i.e. the relationship between the vertices u and v) has to have a degree of membership affected by the memberships of the vertices u and v to the graph.

For the most general definition of a single-value neutrosophic graph, we can remove these restrictions.

It is similar, for the singled-values, to:

- the fuzzy set, where there is the restriction t + f = 1, or t and f are dependent of each other;

- and the intuitionistic fuzzy set, where there is the restriction t + f ≤ 1, similarly t and f are dependent of each other.

While in neutrosophic set, there is no restriction, since t, i, f are considered independent, therefore:

$$t + i + f \leq 3.$$

See my paper about dependence and independence of the components *t, i, f*: *Degree of Dependence and Independence of the (Sub)Components of Fuzzy Set and Neutrosophic Set*, by F. Smarandache, Neutrosophic Sets and Systems, Vol. 11, 95-97, 2016.

Similarly, we can consider the most general case of single-valued neutrosophic graphs, i.e. when the neutrosophic truth-values of the vertices are independent from the neutrosophic truth-values of the edges.

This is just from our everyday life, because:

If John (u) and George (v) are two individuals / vertices in a given set / association (A), and u and v belong each of them in a specific neutrosophic degree respectively (t_u, i_u, f_u) and (t_v, i_v, f_v) to the set A, then the edge uv (meaning the relationship between u and v) is not necessarily dependent on the degrees of appurtenance of u and v to A.

As a particular case, of course we can study too the case when the neutrosophic truth-values of the vertices are dependent of the neutrosophic truth-values of the edges.

You can go further in your Ph D and study the neutrosophic overgraph, neutrosophic undergraph, and neutrosophic offgraph [see *Operators on Single-Valued Neutrosophic Oversets, Neutrosophic Undersets, and Neutrosophic Offsets*, Journal of Mathematics and Informatics, Vol. 5, 63-67, 2016; https://hal.archives-ouvertes.fr/hal-01340833] - which were never studied before.

24 *Paired Structures in Knowledge Representation*

To J. Montero, H. Bustince, C. Franco, J. T. Rodríguez, D. Gómez, M. Pagola, J. Fernandez, E. Barrenechea:

I read with interest your paper "Paired Structures in Knowledge Representation" and the many ideas from it.

*

1) About types of indeterminacy/neutrality I think they depend on the problem to solve. I did into the first attached paper (2013) a refinement of indeterminacy / neutrality within the frame of neutrosophic set and logic.

The number and types of Subindeterminacies depend from a case to another case.

As in neutrosophic set/logic/probability we deal with three components: T, I, F (truth, indeterminacy, falsehood), I have observed that each component can be refined in $T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...$ (I presented this at Universidad Complutense de Madrid in 2014, that Prof. J. Montero and Prof. F. Gallego Lupiañez attended.)

For example, in a voting procedure: T_1 can be the percentage of people from Madrid voting for a candidate, T_2 can be the percentage of people from Salamanca voting for a candidate, etc. While I_1 the percentage of people from Madrid who did not vote, or did a blank vote, or a black vote; etc.

And F_1 percentage of people of people from Madrid voting against a candidate, etc.

Now, if we also consider the voting, but we can take the indeterminacy from a different point of view:

I_1 = percentage of people from Spain who did not vote,

I_2 = percentage of people from Spain that did a blank vote,

I_3 = percentage of people from Spain that did a black vote.

What I meant, it is possible to split Indeterminacy into different ways (into many SubIndeterminacies) depending on what the problem requires us.

*

2) In your paper, it is said that Zhang-Zhang's bipolar model is equivalent to my neutrosophic set. I believe it is not quite exact, since we (Irfan Deli, Mumtaz Ali and Florentin Smarandache, in the paper *Bipolar Neutrosophic Sets and Their Application Based On Multi-Criteria Decision Making Problems*) defined the bipolar neutrosophic set as having $(T^+, T^-, I^+, I^-, F^+, F^-)$ components, where T^+, I^+, F^+ represent the positive components, while T^-, I^-, F^- represent the negative components.

37

25 Neutrosophic Vague Graph

To Said Broumi:

Yes, the vague set can be extended to a neutrosophic vague set, see: *Neutrosophic Vague Set Theory*, by Shawkat Alkhazaleh, *Critical Review*, Volume X, 2015, 29-39, http://fs.gallup.unm.edu/NeutrosophicVagueSetTheory.pdf, and then extended to a neutrosophic vague graph, doing the same interval notation for each of the three components <[T⁻, T⁺], [I⁻, I⁺], [F⁻, F⁺]> for the vertexes and/or edges.

26 Intuitionistic Neutrosophic Set

To Martina Jency:

There are papers on "intuitionistic neutrosophic set" (S. Broumi, F. Smarandache), which consider putting some restrictions on $T(x)$, $I(x)$, $F(x)$; for example:

$\min\{T(x), I(x)\} \leq 0.5$, $\min\{T(x), F(x)\} \leq 0.5$,
and $\min\{I(x), F(x)\} \leq 0.5$,
and therefore $0 \leq T(x) + I(x) + F(x) \leq 2$.

27 Single Valued Neutrosophic Logic

To Martina Jency:

For single valued neutrosophic logic, the sum of the components is:

- $0 \leq t+i+f \leq 3$ when all three components are independent;

- $0 \le t+i+f \le 2$ when two components are dependent, while the third one is independent from them;
- $0 \le t+i+f \le 1$ when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

28 Neutrality Depends of The Problem to Solve

Prof. Javier Montero:

Thanks for your message and your interest in our work, indeed deeply related to your neutrosophic sets. Let me a fast answer right now, but it would be very nice if we can time to put together our visions. I agree with you on that neutrality depends of the problem to solve, but a main claim in our paper (and the previous paper by D. Gomez, J. Montero and H. Bustince: *On the relevance of some families of fuzzy sets*. Fuzzy Sets and Systems 158:2429-2442, 2007), is that we should formalize the differences behind those problems. If we realize that depending on the circumstances, we have different neutralities we should find the way to acknowledge which particular neutrality is suggested in each case. What we propose in this paper is a possible constructive approach. Each word we choose

might represent a specific concept that comes with a specific relational structure. In particular, the term "indeterminacy" might bring certain confusion in my opinion, since there are very different kinds of indeterminacies. Fulfilling both opposites might imply an obvious difficulty to choose among two opposites. But not having information at all, or realizing that none of both opposites hold, also imply difficulties to choose among opposites. But these three cases are essentially different from a knowledge representation point of view. Our objective is precisely to build up a strategy to differentiate and justify all those different neutralities, splitting as you also suggest the big and heterogeneous bag of options in between two opposites.

*

About your example on voting, I can remember that Atanassov in some paper also used a close argument to justify his intuitionistic model. But I think it is a risky example, potentially contaminated with probabilistic arguments.

Anyway, the key issue is how to explain how two opposites create different categories in between and, eventually, a more complex valuation structure.

*

About the equivalence between your model and Zhang-Zhang's model, I am afraid that I took as main reference a perhaps too old paper of yours, where neutrosophy was to my knowledge stated in a much simpler setting. Sorry if I misunderstood your initial

proposal. It is a pity that I did not know about this recent paper of yours, I see that published in the *Proceedings of the 2015 International Conference on Advanced Mechatronic Systems*, Beijing, China, August, 22-24, 2015, but inaccessible for me. Can you send it to me? Thanks, in advance.

29 *Applications of Neutrosophic Set/Logic in Information Fusion*

Prof. Javier Montero, Prof. Francisco Gallego Lupiañez:

¡Hola para todos!

Gracias, Prof. Montero, por su e-mail.

I also CC you colleague and my good researcher in neutrosophic topology, who gave me a ride in Madrid when I visited your university: Prof. Francisco Gallego Lupia *ñ* ez.

My apology for the broken links. They were to the IEEE website, which now ask for payments for each paper. I need to fix them. I attach the first required paper, and a second one which is in the same topic. These papers were for the applications of neutrosophic set / logic in information fusion (where fuzzy logic is also used, besides other special theories of information fusion).

Not only my example with voting, but many other examples one can construct where the indeterminacy / neutrality (actually what is in between opposites, as you say) can be split/refined in different ways. My opinion is that it is not possible to get a unique refinement / splitting

of indeterminacy / neutrality no matter what application one has.

There might be possible, as you said about differences between problems, to consider CLASSES OF PROBLEMS that have the same refinement/splitting of indeterminacy/neutrality. This can work for sure.

But then how many such classes may we have?

A class may depend on the problem to solve and on the type of attribute/action to get into it, i.e.: problem: voting, attribute: cities; then other class, problem: voting, attribute: type of voting (not voting, blank voting, and black voting).

What I mean, even in the frame of the same problem, we may have different attribute to study, and so what we get for the same problem many classes...

¿Que pensas usted?

30 The Law of Included Multiple-Middle

To Dr. Montero et al.:

I come back to your article about types of neutrality/indeterminacy.

A few years ago, I had published a small book of philosophy ("The Law of Included Multiple-Middle").

I said that between the opposites (as in your article) there may be not only one (there is a so-called *Law of Included Middle*) but more Included-Middles, i.e. more neutralities/indeterminacies between the opposites.

Exactly what we all are interested in this research.

Now let's show more examples, as required by Prof. Montero.

1) No middle.

Let's consider the proposition: P be "1+1 = x".

This proposition has two opposites: true for x = 2, and false for x different from 2.

This proposition has no middle / neutrality / indeterminacy.

2) One middle.

Let's consider the proposition:

"In a soccer game, Spain will play versus Germany."

This proposition has two opposites: Spain wins, Spain looses, and one middle: tie game.

3) Two middles.

In the Belnap's quadruple logic, the opposites are T(rue) and F(alse), but there are two middles:

C = contradiction = $T \wedge F$,

and U = unknown = neither T nor F.

4) Three middles.

I defined in 1995 the quintuple logic: the opposites T and F, but there are three middles:

C = contradiction = $T \wedge F$,

U = unknown = neither T nor F,

and G = ignorance = $T \vee F$.

{See my previous paper: Florentin Smarandache*: n-Valued Refined Neutrosophic Logic and Its Applications to Physics*, 2013.}

And so on for refining the indeterminacy / neutrality / middle.

What about if we extend the refinement to the opposites as well? To T (truth) and to F (falsehood) as well.

Let's see a 7-valued neutrosophic logic (with two truths opposed to two falsehoods, and three middles).

As done in the previous n-valued refined neutrosophic logic:

Indeterminacy is similarly refined (split) as above in C, U, G, but T also is refined as T_A = absolute truth (truth in all possible worlds), and T_R = relative truth (truth in at least one world, but not in all worlds), and F is refined as F_A = absolute falsity (falsehood in all possible worlds), and F_R = relative falsity (falsehood in at least one worlds, but not in all worlds).

Where:

$C = (T_A$ or $T_R)$ and $(F_A$ or $F_R)$ (i.e. Contradiction),

U = neither $(T_A$ or $T_R)$ nor $(F_A$ or $F_R)$ (i.e. Undefined),

and $G = (T_A$ or $T_R)$ or $(F_A$ or $F_R)$ (i.e. Ignorance).

31 T, I, F Components

To Prof. Dr. Le Huang Son (Hanoi, Vietnam):

Today, neutrosophic set and neutrosophic logic are the most general and comprehensive logic and respectively sets. They are more flexible and more and more people are starting to apply them in different fields. Indeterminacy makes a difference. Even if intuitionistic fuzzy set let's the possibility to have, when dealing with single values,

1-t-f = indeterminacy, yet when applying any intuitionistic fuzzy operator, the indeterminacy is ignored (i.e. the IF

operators involve only the *T* (membership/truth) and *F* (nonmenbership/falsehood) components, not the *I* (indeterminacy). But the neutrosophic operators involve all three of them: *T, I, F* (so the neutrosophic operators accord a same weight to indeterminacy as to membership / truth and to nonmenbership / falsehood).

And in IFS, when dealing with interval values for *T* and *F*, one does not know exactly how compute what is left as indeterminacy.

*

Actually, when $t + i + f \leq 1$ it is still neutrosophic set, where all three components are dependent and the information is incomplete when $t + i + f < 1$ while when $t + i + f = 1$ the information is complete.

Please check the first lines of the neutrosophic website: http://fs.gallup.unm.edu/neutrosophy.htm .

For single valued neutrosophic logic, the sum of the components is:

- $0 \leq t + i + f \leq 3$ when all three components are independent;
- $0 \leq t + i + f \leq 2$ when two components are dependent, while the third one is independent from them;
- $0 \leq t + i + f \leq 1$ when all three components are dependent.

When three or two of the components *T, I, F* are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components *T, I, F* are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

Always when *t, i, f* are specified, it is neutrosophic set (*neutro* meaning neither true nor false).

Fuzzy set does not allow for the sum of components to be less than 1.

*

In my opinion the name "fuzzy" is not the most appropriate; maybe "neutrosophic dependent incomplete" or something else.

32 Neutrosophic Recommender Systems

To Mumtaz Ali:

It is a good idea to extend the Neutrosophic Recommender Systems and make more or deeper applications to medicine (for the diagnostis).

What about using the refined neutrosophic set (T1, T2, ...; I1. I2, ...; F1, F2, ...)?

33 Picture Fuzzy Relations and Picture Fuzzy Set

To Le, Mumtaz, Cuong, phphong84, Dinh, rtngan, Pham, Said:

Thanks again for the two articles on Picture Fuzzy Relations and Picture Fuzzy Set. I looked over them.

I see the authors use the aggregation operators on all three components *t, i, f,* as in neutrosophic set, not only on *t* (as in fuzzy set), or not only on *t* and *f* (as in intuitionistic

fuzzy set). That's why in my opinion in your future interesting papers where $t + i + t \leq 1$ the better denomination will be Picture Neutrosophic Relation and respectively Picture Neutrosophic Set.

By the way, what is the reason for calling them "picture"? Maybe I miss something herein.

A new possible research will be, if you're interested, when only two components are dependent, while the third one is independent, $i.e. t + i + f \leq 2$.

What do you think about it?

We can consider the case when t and f are dependent, while i independent with respect to both of them. Or when i and f are dependent, and t independent from both of them.

34 Picture Neutrosophic Relation

To Mumtaz Ali, Le Hoang Son, Cuong Bui Cong, et al.:

Picture Fuzzy Set is a special case of Neutrosophic Set (a sub-case of neutrosophic set). In PFS, *t, i, f* depends on each other while in NS, they are independent.

We can also extend NS into some other new theories which can be used in decision making as well as we can construct a huge arithmetic for them.

When "i" indeterminacy is specified, it is part of neutrosophic set.

Even the operators used by authors in PFS and PFR are neutrosophic operators, since as I said before they apply their operators [union, intersection, complement,

etc.] on all three components *t, i, f,* not on two components only (*t* and *f*) as in intuitionistic fuzzy set, and not on one component only (*t*) as in fuzzy set.

I think in the paper which was not published on Picture Fuzzy Relation, the authors should replace the syntagm Picture Fuzzy Relation by "Picture Neutrosophic Relation" (or 3-Dependent Neutrosophic Relation, or something similar).

We can publish their paper in *Neutrosophic Sets and Systems* or in *Critical Reviews* if they wish.

For the published paper, it is nothing to do, but in the future I think the authors should use the right definition (Neutrosophic instead of Fuzzy).

35 *Refining the Neutrosophic Set (and of Picture Fuzzy Set)*

To: Le, Mumtaz, Cuong, phphong84, Dinh, rtngan, Pham, Said:

I read your example with voting, how to refine it.

In general, it is possible to even more refine it, as:

$t_1, t_2, ...; \ i_1, i_2, ...; \ f_1, f_2, ... $.

The operators used (min/max/max or max/min/min, etc.) are similar to the neutrosophic operators used in the book on Interval Neutrosophic set and Logic.

There are other types of neutrosophic operatos as well:

- $t_1 t_2, \ i_1+i_2 - i_1 i_2, \ f_1+f_2-f_1 f_2$, for intersection,
- $t_1+t_1-t_1 t_2, \ i_1 i_2, \ f_1 f_2$, for union, etc.

You can exercise various operators in your PFSs.

36 *Neutrosophic Set and Picture Fuzzy Set*

Le Huang Son wrote to Prof. Smarandache, Prof. Bui Cong Cuong, et al.

Hereinafter, I would like to officially introduce two pioneers: *Prof. Florentin Smarandache* (University of New Mexico, USA) who invented "Neutrosophic Set" that has been widely investigated from 2002. Till now, there are approximately 2000 papers about NS; *Prof. Bui Cong Cuong* (Vietnam Institute of Mathematics) who presented the notion of "Picture Fuzzy Set" from 2013. As you know, there are several common charateristics between two notions, for example:

A neutrosophic set is defined as
A = {(x, T(x), I(x), F(x))}, where $T + I + F \leq 3$; *T, I, F* are functions in [0, 1].

While a picture fuzzy set is
B = {(x, u, e, v)} satistifying $u + e + v \leq 1$; *u, e, v* are CRISP values in [0, 1].

Both sets are generalizations of Intuitionistic Fuzzy Set. I don't want to discuss further about their coincidence as I believe it would be more suitable when Prof. Smarandache comes to Vietnam in May 2016. We will clarify any detail.

Information about his visit was given in my last email. Should you have any comment or idea, please let me know.

37 Neutrosophic 2-Dependent Components

To Said Broumi:

Nobody has done research or examples in this case, i.e. if for single valued components *t, i* are dependent then $0 \leq t + i \leq 1$, and if *f* is independent from them, then $0 \leq f \leq 1$, therefore per total: $0 \leq t + i + f \leq 2$. *t* and *i* may be dependent when they are for example given by the same source. Or we may consider the case when *t* and *f* are dependent, while *i* is independent from them.

38 I-neutrosophic Fuzzy Subgroup/Group

To Mumtaz Ali:

I checked the paper and the definitions are direct extensions from Fuzzy Subgroupoids / Subgroups to Neutrosophic Fuzzy Subgroupoids / Subgroups.

What about considering: Let G be a groupoid.

We define the groupoid neutrosophic partial order:

$a_1 + b_1 I \geq a_2 + b_2 I$ if $a_1 \geq a_2$ and $b_1 \geq b_2$.

Let μ: G -> {a+bI, where a, b $\in$ [0, 1]},

with $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, for all x, y $\in$ G, then μ is a I-neutrosophic fuzzy subgroup.

Similarly: Let G be a group.

We similarly define the group neutrosophic partial order:

$a_1 + b_1 I \geq a_2 + b_2 I$ if $a_1 \geq a_2$ and $b_1 \geq b_2$.

Let μ: G -> {a+bI, where a, b $\in$ [0, 1]},

with $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, and $\mu(x^{-1}) \geq \mu(x)$, for all x, y in G, then μ is a I-neutrosophic fuzzy group.

39 *Refined Complex Neutrosophic Set*

To Mumtaz Ali:

We can extend the complex neutrosophic set to the *refined complex neutrosophic set*, using T_1, T_2, ...; I_1, I_2, ...; and F_1, F_2, ...

40 *Neutrosophy in Medical Diagnosis*

To Mumtaz Ali, Nguyen Minh, and Le Huang Son:

I read your interesting paper, which introduces for the first time the Neutrosophic Recommender System.

I also saw the nice applications into the medical diagnosis.

I think it's possible to do another approach using the Neutrosophic Implication Operator [see some examples of classes of neutrosophic implications in the attached book at pp. 79-83 for neutrosophic numerical implications, and at p. 177 for neutrosophic literal implication].

I mean, we can map $R(p_i, s_j) \wedge R(s_j, d_k)$ into $R(p_i, d_k)$ using the neutrosophic implication: meaning the neutrosophic mapping of the relationships between

(patient p_i with symptom s_j) $\wedge$ (symptom s_j with disease d_k) into the relationship (patient p_i with disease d_k), where $\wedge$ is the neutrosphic conjunction.

Numerical example: One has the following two neutrosophic relationships:

(John, temperature) = (0.6, 0.2, 0.1),

and   (temperature, viral fever) = (0.7, 0.1, 0.3).

Actually, we need only the neutrosophic conjunction:
(John, temperature) $\wedge$ (temperature, viral fever) =
(John, viral fever).

Hence:

(0.6, 0.2, 0.1) $\wedge$ (0.7, 0.1, 0.3) =

(min{0.6, 0.7}, max{0.2, 0.1}, max{0.1, 0.3}) = (0.6, 0.2, 0.3).

*

Most papers on neutrosophic medical diagnosis use the implication operator to derive the relationship between patients and diseases. Nonetheless, we are on a different way - using a non-linear model to find the relationship from the most analogous patients. But your ideas are still helpful. But note that in practical medical diagnosis, we have never got the relation between (temperature, viral fever) because it depends on each clinician. In order to fix (or accommodate) it, what about if instead of single value neutrosophic set we take interval valued neutrosophic set. I mean instead for example of (40 grade temperature, viral fever), we can go by ([30-50], viral fever). May it work?

*

If we have both tables (patients, symptoms) and part of (patients, diseases), what should we do to increase the accuracy of diagnosis? If we have (John, symptom1), (John, viral fever) we can associate (symptom1, viral fever). If (George, symptom2) and (George, viral fever), then we associate (symptom2, viral fever), etc. So, in my opinion, we associate all symptoms from all patients to the same disease.

Le Huang Son:

This is obvious to use interval or "fuzzy of fuzzy" for the relation (temperature, viral fever) but note that this kind of data is not available in practical. What we have is medical records of patients showing their symptoms and the patient history telling the previous disease that he/she acquire.

Florentin Smarandache:

Probably considering various patients for the same disease, their symptoms are not exactly the same. There is some variation between a symptom of a patient and a symptom of another patient both having the same disease.

Le Huang Son:

Note that clinicians are not mathematician and informaticians. They do not have any idea of the numbers and their meaning.

Florentin Smarandache:

Then we may work with linguistic variables (low temperature, normal temperature, high temperature.

Also, many algorithms are implemented in computer programs that are (or should be) friendly to users.

Le Huang Son:

The point is: we have (John, symptom1) but do not have any record named John in (patients, diseases) table since he is a new one. How can we do?

Florentin Smarandache:

I was talking about the patients we have already, so we can compare their symptoms for the same disease.

When a new patient comes in, we find his syndromes, then we match them versus an electronic database (dataset) of symptoms-diseases and we can see what disease is closest to his/her symptoms using a computer program. [As for example police does for matching the fingerprints of criminals versus a database of fingerprints.]

These database or course is updated and increased during time.

41 Neutrosophic Infinity

Florentin Smarandache:

The idea of putting $1/0$, $2/0$, etc. as subindeterminacies respectively I_o^1, I_o^2, etc. in Z_n within the neutrosophic frame is marvelous.

You're right. It makes sense, because:

In Z_4 one has:

$2 / 2$ = zerodivisor / zerodivisor = indeterminacy

and $0 / 2$ = zero / zerodivisor = indeterminacy.

Actually, a *zerodivisor* is a *neutrosophic zero*.

W. B. Vasantha Kandasamy:

For only now I am realizing that neutrosopy has zero of its own, in fact infinite number of such zeros, which have given multi-dimension to neutrosophy: one in real, versus infinite in neutrosophic field.

So some more new notions for our books everything makes us more mathematically productive.

Florentin Smarandache:
Let's consider two classical functions: f(x) and g(x) defined from R → R, where R = the set of real numbers.

Then, the neutrosophic division of functions, is defined as follows:

f(x) / g(x) = f(x)/g(x), if g(x) is not equal to zero, and $I_o^{f(x)}$ = indeterminacy of the form r/o, where r = real number, when g(x) = o.

Actually, we can initiate a new type of mathematics on indeterminacy. What do you think?

Dr. W. B. Vasantha Kandasamy:
This idea is very bright, we will do.

Florentin Smarandache:
o^o = indeterminate.

We consider the power function f: R→R, f(x) = x^x, where for x > o, f(x) is determinate; for x = o, f(o) = o^o → 1, but for x < o, f(x) is a complex number; how should we interpret this third case?

Can we still say that, for x < o, f(x) is indeterminate since f(x) is not in R?

W. B. Vasantha Kandasamy:
For finite complex number, one has two situations:

One, if it is a unit in that case is normal.

Second, if it not a unit, we will have MOD finite complex natural neutrosophic numbers so this idea is fine, we will work.

Florentin Smarandache:

There are seven undefined operations in calculus:

- subtraction: $\infty - \infty$
- multiplication: $0 \times \infty$
- division: $0/0$ (we did already), ∞/∞
- power: 0^0, ∞^0, 1^∞

Can we design somehow mathematical entities that involve these undefined operations that can be used as indeterminacies? Any idea how?

W. B. Vasantha Kandasamy:

Sure, this is also a fantastic idea, we will do.

Florentin Smarandache:

It's marvelous to get something about infinity within the frame of neutrosophics.

I'll think how to insert the infinity into the frame of neutrosophics.

For example, the notion of 'neutrosophicinfinity'.

W. B. Vasantha Kandasamy:

Sure, the calculus which is full of flaws needs some basic reformation.

Florentin Smarandache:

Dr. Vasantha, you might know this attached book. It is only an introduction in neutrosophic calculus. I did not say anything on infinity from a neutrosophic point of view.

Neutrosophic infinity may be an infinity with some indeterminacy (meaning for example: that we are not sure if it is infinity or not?)... Or what other indeterminacy to insert into infinity?

*

Let's continue the introduction of the neutrosophic infinity. I had some questions I did not understand. For example: in $(Z_8, \times)$ with respect to the common multiplication there are three zerodivisors: 2, 4, 6.

Don't we then have four neutrosophic indeterminacies:

- $0/0$, $1/0$, $2/0$, ..., $7/0 = I_0^8 =$ neutrosophic indeterminacy 0;
- $0/2$, $1/2$, $2/2$, ..., $7/2 = I_2^8 =$ neutrosophic indeterminacy 2;
- $0/4$, $1/4$, $2/4$, ..., $7/4 = I_4^8 =$ neutrosophic indeterminacy 4;
- $0/6$, $1/6$, $2/6$, ..., $7/6 = I_6^8 =$ neutrosophic indeterminacy 6 ?

Then $Z_8^I = \{0, 1, 2, ..., 7; I_0^8, I_2^8, I_4^8, I_6^8\}$

Now,

- the inverse of I_0^8, with respect to the common multiplication, in Z_8^I should be NeutrosophicInfinity$_0^8$.

Similarly:

- the inverse of $I_2{}^8$, with respect to the common multiplication, in Z_8^I should be NeutrosophicInfinity$_2{}^8$.
- the inverse of $I_4{}^8$, with respect to the common multiplication, in Z_8^I should be NeutrosophicInfinity$_4{}^8$.
- the inverse of $I_6{}^8$, with respect to the common multiplication, in Z_8^I should be NeutrosophicInfinity$_6{}^8$.

So, we have six neutrosophic infinities in Z_8^I.

Are these neutrosophicinfinities equal, or different?

W. B. Vasantha Kandasamy:

Only one neutrosophic infinity.

Florentin Smarandache:

Why should we consider all of them as one?

See zero divisor are different but we have only one neutrosophic zero for this Z_8. Also, what properties can we have about them? What kind of neutrosophic algebraic structure becomes Z_8^I ?

It will be semigroup under +, and a semigroup under $\times$, and only a semiring under + and $\times$.

n/o in Z_8 is a "natural neutrosophic zero", but the denomination "natural neutrosophic zero" is not very good... it is confusion.

For example, 0+0I is a natural neutrosophic zero...

I know this was used in the previous MOD books.

Better we should call it "neutrosophic zero indeterminate" since it is n/0, while $n/2$, $n/4$, $n/6$ in Z_8 should be called "neutrosophic zerodivisor indeterminate".

Now the definitions clearly tell us what they are and how they were.

We should only add these two defintions at the beginning of the last MOD book.

In $Z_4 = \{0, 1, 2, 3\}$ one has with respect to the NZI = neutrosophic zero indeterminate = $1/0 = 2/0 = 3/0$.

And one NZDI = Neutrosophic ZeroDivisor Indeterminate = $0/2 = 1/2 = 2/2 = 3/2$.

Now, the inverse of NZI be by notation NZI' which is actually NZI^{-1} that is NeutrosophicInfinity$_1$, and the inverse of NZDI' which is actually $NZDI^{-1}$ that is NeutrosophicInfinity$_2$.

Therefore, the neutrosophic extension of Z_4 is now:

$Z_4^I = \{ 0, 1, 2, 3;$ NZI, NZDI; NZI', NZDI' $\}$

NZI $\times$ NZDI = NZI

$3 \times$ NZI = NZI

$3 \times$ NDZI = NDZI

Do we have NZI $\times$ NZI' = 1 and NZDI $\times$ NZDI' = 1 ?

What is NZI $\times$ NZDI' = ?

What structure did we get?

- Neutrosophic zero indeterminate is $k/0$ in Z_n.
- Neutrosophic zerodivisor indeterminate is n/zerodivisor in Z_n.
- And neutrosophic infinity is the inverse of each of them...

42 Neutrosophic Multigraphs with Multiloops

To Said Broumi:

Try to study also the multigraph, which is a graph where more than one edge joins two vertices, called multiple edges, or parallel edges. And extend them to neutrosophic multigraphs. Also, go even further, and consider multi-loops too for the same vertex into the neutrosophic multigraph with multiloops.

43 Neutrosophic Quadruple Numbers, Generalized Pythagorean Neutrosophic Set

To Mumtaz Ali, Le Hoang Son, Luu Dat, and Nguen Van Minh:

I have defined theoretically numbers of the following form:

$$a + bT + cI + dF,$$

called neutrosophic quadruple numbers (*a, b, c, d* are real numbers; but we can also consider the case when they are complex numbers), where T = truth/membership, I = indeterminacy, and F = false/nonmembership.

I did not look for applications, but if you have such ideas of applications it would be innovatory.

Mumtaz is a specialist in algebraic structures. We may try to build some neutrosophic quadruple algebraic structures, by defining algebraic laws on such numbers.

We may also develop neutrosophic algebraic structures on sets of neutrosophic quaternions, neutrosophic octonions, neutrosophic dual numbers, etc.

Mumtaz Ali:

A Spheroid Neutrosophic Set (also called Generalized Pythagorean Neutrosophic Set) if $T^2+I^2+F^2 \leq 3$.

Also, we need to explain that it is only 1/8 of the sphere $T^2 + I^2 + F^2 = 3$.

Also, we need to explain that it is only 1/8 of the sphere $T^2 + I^2 + F^2 = 3$.

It is no problem because the sphere's radius is squareroot (3).

Because even the Pythagorean Fuzzy Set
$T^2 + F^2 = 1$ is only 1/4 of the circle.

Florentin Smarandache:

Yes, $T^2+I^2+F^2 \leq 3$ is 1/8 of the sphere of radius $\sqrt{3}$, because: all T, I, F are $\geq$ 0.

The sphere is divided into 8 equal spheroidal sectors, i.e.:

1) $T \geq 0, I \geq 0, F \geq 0$ {this is our neutrosophic set/logic};
2) $T \geq 0, I \geq 0, F \leq 0$;
3) $T \geq 0, I \leq 0, F \geq 0$;
4) $T \leq 0, I \geq 0, F \geq 0$;
5) $T \geq 0, I \leq 0, F \leq 0$;
6) $T \leq 0, I \geq 0, F \leq 0$;
7) $T \leq 0, I \leq 0, F \geq 0$;
8) $T \leq 0, I \leq 0, F \leq 0$.

You can include the above proof, since the 8 spheroidal sectors are equal, and since in a sphere centered

in (0, 0, 0) whose coordinate axes are (T), (I), and (F), and its radius is $\sqrt{3}$, each component T, I, F is in between $-\sqrt{3}$ and $+\sqrt{3}$, but since in neutrosophic set T, I, F ≥ 0, we take only one spheroidal sector.

We can also extend to bipolar spheroidal neutrosophic set, where we have positive and negative values for T, I, F.

In higher geometry, we can consider the hyper-sphere, which is a sphere of dimension $n \geq 3$, i.e. $x_1^2 + x_2^2 + \ldots + x_n^2 \leq r^2$, where r = radius, and $x_1, x_2, \ldots, x_n$, $n \geq 3$, the coordinate axes.

So it is okay to consider for a neutrosophic set a complex form of dimension 4 (or of any dimension).

Mumtaz Ali:

I just want to extend the Pythagorean fuzzy set to neutrosophic set. A Pythagorean fuzzy set is an intuitionistic fuzzy set which satisfies the condition of $u^2 + v^2 \leq 1$, where u and v are membership and non-membership functions.

I want to apply this condition in neutrosophic set to define spheroid neutrosophic set (generalized pythagorean fuzzy set) as $T^2 + I^2 + F^2 \leq 3$ seems to be 1/8 of the sphere.

To Mumtaz Ali:

We should think at some real application, or real example from our everyday life, for this neutrosophic

quadruple. Indeed, a new paper is needed for a+bT+cI+dF quadruple. We should think at some real application, or real example from our everyday life for this neutrosophic quadruple number. [I thought you connected it to the complex neutrosophic set... My misunderstanding...]

*

The second question. See three interpretations:

1) T, I, F can be interpreted in a 3-dimensional space, whose coordinate REAL axes are T (alike X), F (alike Y), and I (alike Z). We get a neutrosophic cube (I told it to you before) for $0 \leq T, I, F \leq 1$.

This is a simple interpretation.

2) Another one will be to consider dimension 6, i.e. T represented by a COMPLEX axis (with real part truth and imaginary part truth), similarly F represented by a COMPLEX axis (with real part falsehood and imaginary part falsehood), and finally I represented by a COMPLEX axis (with real part indeterminacy and imaginary part indeterminacy).

This is the most sophisticated, but the most accurate.

What about this interpretation?

3) We may try to invent a *triple complex number* of the form a+bi+cj, where a, b, c are real numbers, and
$i^2 = -1$, $j^4 = -1$... {More thinking needed about how to define "j".}

Then a = membership, b = nonmembership, and c = indeterminacy. It is dimension 3 as you want for neutrosophic complex set.

Something like that. Any suggestion?

Florentin Smarandache to Mumtaz Ali:

We can do a paper where (T₁+jT₂, I₁+jI₂, F₁+jF₂) are just complex numbers.

Mumtaz Ali:

We can use them as (T+jF, I+jI, F+jT). Since if T is along x-axis, then F must be along y-axis, so we keep them as T+jF. Similar case for F+jT. But what should be I+jI interpretation?

Florentin Smarandache:

We may consider the indeterminacy "I" split, for example, into uncertainty & contradiction, i.e. I+jI means Uncertainty+jContradiction.

Or U+jC, where U = uncertainty and C = contradiction.

We can spit the indeterminacy in many different ways, depending on the problem (see my paper on Refined Indeterminacy).

Florentin Smarandache:

For $<T_1+jT_2, I_1+jI_2, F_1+jF_2>$ we can consider them as refined components, i.e. T_1 is a type of truth and T_2 another type of truth; similarly for I_1, I_2 and respectively F_1, F_2; where $j = \sqrt{-1}$.

See my refined indeterminacy, and also the refined all components T, I, F papers.

Mumtaz Ali:

It is reduced to the following form:

(T, I, F) +j(T, I,F) as we have (T+jF, I+jI, F+jT). It works like complex numbers as you see. We can work with it but I don't see any practical situation. What do you think?
Florentin Smarandache:

I think we should mention it theoretically anyway, because maybe later applications will be found. It happened in science that some theoretical ideas became practical a century later!

Mumtaz Ali:

I am more interested in 3 dimensional idea of neutrosophic set like Ti+Ij + Fk where T, I, F are truth membership, indeterminacy and falsehood membership function while i, j , k are unit vectors.

Florentin Smarandache:

Firstly, I'd rather use the notations: Th+Ij+Fk, in order to avoid the confusion of "i" and "I".

Yes, this is connected with the Neutrosophic Cube I sent you before (see it again attached).

I.e. <T, I, F>, which is a point P inside or on the neutrosophic cube, can also be written as a vector: -> OP = Th+Ij+Fk, where h, j, k are unit vectors, and O is the origin of the 3D-Cartesian system of coordinates of axes x, y, z.

$|Th+Ij+Fj|$ = squareroot($T^2+I^2+F^2$) which is the magnitude of vector OP, but also the distance between points O and P.

All operations [additions/subtractions, also scalar and vector multiplications] of $T_1h+I_1J+F_1K$ (corresponding to point P_1 inside the neutrosophic cube) with $T_2h+I_2J+F_2K$ (corresponding to point P_2 inside the neutrosophic cube) can be considered as operations on the vectors OP_1 and OP_2. The result of a vectorial operation has to be also inside the neutrosophic cube.

Mumtaz Ali:

More interesting idea will be of quaternion number representation of neutrosophic set. The only problem here is that Quaternion number is 4 dimensional while neutrosophic set is 3 dimensional.

Florentin Smarandache:

First of all, why do you want to stick with 3 dimensions? Why 4 dimensions are not okay?

Do you want to consider neutrosophic quaternions of the form:

$a+T_h+I_j+F_k$? What "a" should mean?

44 Neutrosophic Quadruple Structures

Let $Q = \{a+bT+cI+dF$, where a, b, c, d are real numbers, and T = truth, I = indeterminacy, F = falsehood}.

The preference law among T, I, F is defined as $T < I < F$, which means that when one multiplies two of them the result is equal to the greater one that absorbs the smaller one, i.e. TT = T (T absorbs itself), TI = I (because I

is bigger than T, so I absorbs T), TF = F (because F is bigger than T, so F absorbs T), II = I (I absorbs itself), IF = F (because F is bigger than I, so F absorbs I), FF = F (F absorbs itself).

One defines the laws: + and $\times$.

$(a_1+b_1T+c_1I+d_1F) + (a_2+b_2T+c_2I+d_2F) =$

$(a_1+a_2)T + (b_1+b_2)I + (c_1+c_2)F.$

$(Q, +)$ is a commutative group.

Then:

$(a_1+b_1T+c_1I+d_1F) \times (a_2+b_2T+c_2I+d_2F) =$

$a_1a_2 + (a_1b_2+a_2b_1+b_1b_2)T + (a_1c_2+a_2c_1+b_1c_2+b_2c_1+c_1c_2)I + (a_1d_2+a_2d_1+b_1d_2+b_2d_1+c_1d_2+c_2d_1+d_1d_2)F$

$(Q, \times)$ is well defined, associative, commutative, and has a neutral element equal to 1; but it does not have an inverse element. $(Q, \times)$ is a commutative monoid.

45 Refined Single-Valued Neutrosophic Sets

To Dr. Jun Ye, Dr. P. Majumdar and Dr. U. Samanta:

Your similarity method of single-valued neutrosophic sets A and B to *refined* single-valued neutrosophic sets of the form (Smarandache, 2013): t_1, t_2, ...t_j; i_1, i_2, ..., i_k; f_1, f_2, ..., f_l, with $j + k + l = n > 3$.

Florentin Smarandache, *n-Valued Refined Neutrosophic Logic and Its Applications in Physics*, Progress in Physics, 143-146, Vol. 4, 2013;
http://fs.gallup.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf

To Jun Ye:

Let U be the universe of discourse, and A and B be two (non-refined) single-valued neutrosophic sets,

$$A = \{< x_i, T_A(x_i), I_A(x_i), F_A(x_i) >, x_i \in U\}$$

and $B = \{< x_i, T_B(x_i), I_B(x_i), F_B(x_i) >, x_i \in U\}$.

Majumdar & Samanta's Similarity Method of two (non-refined) single-valued neutrosophic sets A and B is:

$$S_{MS}(A,B) = \frac{\sum_{i=1}^{n}[\min\{T_A(x_i), T_B(x_i)\} + \min\{I_A(x_i), I_B(x_i)\} + \min\{F_A(x_i), F_B(x_i)\}]}{\sum_{i=1}^{n}[\max\{T_A(x_i), T_B(x_i)\} + \max\{I_A(x_i), I_B(x_i)\} + \max\{F_A(x_i), F_B(x_i)\}]}$$

Ye's Similarity Method of two (non-refined) single-valued neutrosophic sets A and B is:

$$S_{Ye}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{\min\{T_A(x_i), T_B(x_i)\} + \min\{I_A(x_i), I_B(x_i)\} + \min\{F_A(x_i), F_B(x_i)\}}{\max\{T_A(x_i), T_B(x_i)\} + \max\{I_A(x_i), I_B(x_i)\} + \max\{F_A(x_i), F_B(x_i)\}}$$

Smarandache & Ye's Similarity Methods of Refined single-valued neutrosophic sets is below.

Let A_{ref} and B_{ref} be two refined single-valued neutrosophic sets defined as follows:

$$A_{ref} = \{< x_i, T_{1A}(x_i), T_{2A}(x_i), ..., T_{pA}(x_i); I_{1A}(x_i), I_{2A}(x_i), ..., I_{rA}(x_i);$$
$$F_{1A}(x_i), F_{2A}(x_i), ..., F_{sA}(x_i) >, x_i \in U\},$$
$$B_{ref} = \{< x_i, T_{1B}(x_i), T_{2B}(x_i), ..., T_{pB}(x_i); I_{1B}(x_i), I_{2B}(x_i), ..., I_{rB}(x_i);$$
$$F_{1B}(x_i), F_{2B}(x_i), ..., F_{sB}(x_i) >, x_i \in U\}, p + r + s \geq 3,$$

where p, r, and s are positive integers, and all T_{jA}, I_{kA}, F_{lA} and T_{jB}, I_{kB}, F_{lB} belong to [0, 1].

Then, we extend Majumdar & Samanta's Similarity Method for two refined single-valued refined neutrosophic sets A_{ref} and B_{ref} as follows:

$$S^{ref}_{SY-MS}(A,B) = \frac{\sum_{i=1}^{n}[\sum_{j=1}^{p}\min\{T_{jA}(x_i),T_{jB}(x_i)\} + \sum_{k=1}^{r}\min\{I_{kA}(x_i),I_{kB}(x_i)\} + \sum_{l=1}^{s}\min\{F_{lA}(x_i),F_{lB}(x_i)\}]}{\sum_{i=1}^{n}[\sum_{j=1}^{p}\max\{T_{jA}(x_i),T_{jB}(x_i)\} + \sum_{k=1}^{r}\max\{I_{kA}(x_i),I_{kB}(x_i)\} + \sum_{l=1}^{s}\max F_{lA}(x_i),F_{lB}(x_i)\}]}$$

And we also extend Ye's Similarity Method for refined single-valued refined neutrosophic sets A$_{ref}$ and B$_{ref}$ as follows:

$$S^{ref}_{SY}(A,B) = \frac{1}{n}\sum_{i=1}^{n}\frac{\sum_{j=1}^{p}\min\{T_{jA}(x_i),T_{jB}(x_i)\} + \sum_{k=1}^{r}\min\{I_{kA}(x_i),I_{kB}(x_i)\} + \sum_{l=1}^{s}\min\{F_{lA}(x_i),F_{lB}(x_i)\}}{\sum_{j=1}^{p}\max T_{jA}(x_i),T_{jB}(x_i)\} + \sum_{k=1}^{r}\max\{I_{kA}(x_i),I_{kB}(x_i)\} + \sum_{l=1}^{s}\max\{F_{lA}(x_i),F_{lB}(x_i)\}}$$

References:

Majumdar, P. and Samanta, S. K. (2014) On similarity and entropy of neutrosophic sets, Journal of Intelligent and Fuzzy Systems 26, 1245-1252.

Ye, J. (2016) Single valued neutrosophic clustering algorithms based on similarity measures, mss. Submitted to Journal of Classification.

Jun Ye:

Due to your new neutrosophic theory, I can apply it to engineering applications.

As next wok, I am going to apply the neutrosophic probability to rock mechanics.

46 *Strong and Regular Unmatter*

Dmitri Rabounski:

Anti-particle in physics means a particle which has one or more opposite properties to its "original particle

kind". It can be electric charge, color or fragrance (for quarks), or everything else. Yes, a particle and its anti-particle annihilate into gamma-quanta if meet each other.

That is, in physics, even if one property of a particle has opposite sign to its original state, this particle is anti-particle, and it annihilates with its original particle.

This formulation may be mistaken with the neutrosophic <antiA>, which is strong opposite to the original particle kind. I suppose that the <antiA> state is the ultimate case of anti-particles, as we stated in our common book "Neutrosophic Relativity" (I do not remember where in exact). In any case, those persons who refer <A>, <neutA>, <anti-A> to as "matter", "unmatter", and "anti-matter" are absolutely right.

Following this way, in analogy to anti-matter as the ultimate case of anti-particles in physics, we can extend the term unmatter: "strong unmatter" is that where all properties of a substance or a field are unmatter, and "regular unmatter" where just one of the properties of it satisfies the unmatter.

*

To Dmitri Rabounski:

In *Progress in Physics* (2013) I published a small paper about refinement of neutrosophic logic.

Hence, <A>, <neutA> and <antiA> can be split into:

<A_1>, <A_2>, ...; <$neutA_1$>, <$neutA_2$>, ...; <$antiA_1$>, <$antiA_2$>, ...; therefore more types of matter, more types of unmatter, and more types of antimatter.

To Mumtaz Ali:

The advantage of using nonstandard number is that it makes a distinction between absolute truth / indeterminacy / falsehood and relative truth / indeterminacy / falsehood: "T = 1+" means ABSOLUTE TRUTH, i.e. truth in all possible worlds, while "T = 1"means RELATIVE TRUTH, i.e. truth in at least one world.

Similarly for "I = 1+" and for "F = 1+" and similarly for T, I, F equal "-0".

Mostly in philosophy it can be used, not in technique.

47 Trapezoidal Neutrosophic Cognitive Maps

To W. B. Vasantha Kandasamy:

Dr. Arthur Rajkumar from Chennai [maybe you know him] proposed the *Triangular Neutrosophic Cognitive Maps*. I did not get his paper.

Would we then extend the NCM to *Trapezoidal Neutrosophic Cognitive Maps*?

48 Neutrosophic Soft Linear Code

To Mumtaz Ali & W. B. Vasantha Kandasamy:

The main distinction between linear code and soft linear code is that for the soft linear code each soft codeword has some flavors, i.e. each soft codeword is characterized by some attributes, while the non-soft codewords are characterized by no attributes.

Then one can manipulate the attributes of a soft codewords, for example an attribute "a₁" can some some attribute that 'trick' the code hackers, or may be chances that the soft codeword has a smaller chance to belong to the message (i.e. included just to deceive the hackers). And so on.

What do you think, Dr. W. B. Vasantha Kandasamy and Mr. Mumtaz Ali, if we associate such attributes to the soft code? Maybe we can associate just a neutrosophic component to a codewords (i.e. t = chance that the word is valuable, f = chance that the codeword is not valuable, i = indeterminacy)...

*

We can consider the neutrosophic soft linear code.

I have the idea that we can consider the "neutrosophic cut" for the codewords.

Let's consider a simple attribute set A = {good, bad}.

{Surely, more refined attribute set can be designed - if we want.}

Then the codewards that have the "bad" attribute are rejected from the message, while the codewards that have the "good" attribute in a percentage $t \geq 0.7$ and $i \leq 0.3$ and $f \leq 0,3$ are accepted.

So, the neutrosophic cut used was: $t \geq 0.7$, $i \leq 0.3$, $f \leq 0.3$.

Of course, other numbers we can use for the neutrosophic cut.

49 *Complex Neutrosophic Fuzzy Classes*

To Mumtaz Ali:

Complex Intuitionistic Fuzzy Classes can be straightforward extended to Complex Neutrosophic Fuzzy Classes by adding the Pure Complex Neutrosophic Degree of Indeterminacy, or by considering the Complex Neutrosophic Set extended from a neutrosophic set to a neutrosophic set of neutrosophic sets.

50 *Singled-Value Neutrosophic Graph Theory*

To Said Broumi:

The influence coefficient of a single-valued neutrosophic number (t, i, f) as defined by Ridvan Sahin is logically and intuitively correct as (1/3)(t+ 1-i + 1-f) because, as he very well said, the true degree (t) proves a positive impact, while indeterminacy (i) and falsity (f) prove negative impacts in the relationship, that's why one take the opposites 1-i and respectively 1-f, and then one makes the average of all positive impacts t, 1-i, and 1-f.

In the singled-value neutrosophic graph theory if the edge between vertices α_1 and α_2 has the neutrosophic value (t, i, f), then the influence coefficient (1/3)(t+ 1-i + 1-f) should represent the influence coefficient between the vertices α_1 and α_2.

51 Applications of Neutrosophic Logic

Victor Christianto:

I would advise to study neutrosophic logic for applications such as conflict resolution, reconciliation, and religious tolerance.

*

To V. Christianto's group:

In the modern logic the excluded middle by Aristotle has been replaced by included middle: <A>, <neutA>, and <antiA>, where <neutA> is the neutral part, i.e. neither <A> nor its opposite <antiA>, but situated in the middle (neutral part of the opposites). So, the dialectics based on the dynamics of opposites <A> and <antiA> has been extended to neutrosophy, which means dynamics of all three parts: <A>, <antiA>, and <neutA>, which better reflect our reality. Because, for example, if two countries go to war, <A> vs. <antiA>, some neutral countries may interfere and ally with one or another side, or may help or entangle one or the other side. Then further, the law of included middle has been extended to the Law of Multiple-Included Middle, i.e. $<neutA_1>$, $<neutA_2>$, ..., $<neutA_n>$.

*

To Dr. Kanda:

Neutrosophic Logic does not solve contradictions or conflicts, it just measures their truth-values. It shows that when the sum of components is > 1, one has conflicting cases. Similarly, as Boolean Logic or Fuzzy Logic that do not solve things, but measure their truth-values.

52 Single-Valued Complex Neutrosophic Set, Interval Complex Neutrosophic Set, Subset Complex Neutrosophic Set

To Mumtaz Ali, Le Hoang Son and Luu Quoc Dat:

Extending the definition. I propose to extend the definition of complex neutrosophic set we did before (when we considered the real parts and imaginary parts of all T, I, F neutrosophic components as crisp numbers at the beginning, then later as interval-values), to its largest one (as I did for neutrosophic set in 1995-1998): i.e. to consider that the real parts and imaginary parts of all T, I, F neutrosophic components are standard or non-standard subsets of the unitary non-standard interval $]^-0, 1^+[$.

This is theoretical. Then we justify that in scientific and technical applications we use the real parts and imaginary parts of all T, I, F neutrosophic components as only standard subsets of the unitary standard interval $[0, 1]$.

Therefore, we have: Single-Valued Complex Neutrosophic Set, Interval Complex Neutrosophic Set, and Subset Complex Neutrosophic Set.

53 Complex Neutrosophic Logic

To Mumtaz Ali, Le Hoang Son and Luu Quoc Dat:

Analogy to fuzzy environment. For constructing the Complex Neutrosophic Logic, we may take a look at Complex Fuzzy Logic (if any) and then go by analogy.

Actually, the aggregation operators for Complex Neutrosophic Logic will be similar to those of Complex Neutrosophic Set:

- AND as intersection
- OR as union
- NEGATION as complement   Etc.

54 Refined Subset Complex Neutrosophic Logic

To Mumtaz Ali, Le Hoang Son and Luu Quoc Dat:
I come back with the idea of making refinement (T_j, I_k, F_l) for $j + k + l = n \geq 4$, j, k, l $\geq$ 1, and T_j, I_k, F_l are complex numbers whose real parts and imaginary parts are subsets of [0, 1]. {Again, the largest possible extension of the Complex Neutrosophic Logic.}

55 Neutrosophics in Computer Vision

To Said Broumi (Morocco), Mumtaz Ali (Pakistan), W. B. Vasantha Kandasamy (India):
Can we use the neutrosophics in Computer Vision?

56 Neutrosophic Thick Function

To Mumtaz Ali:
Since you asked me about e^S, where "e" is Euler's constant and S is a set, please see my book on *Neutrosophic Precalculus and Neutrosophic Calculus* (2015). At pages 14 and 28 you see the general definition of the Neutrosophic Function (i.e. functions that have some indeterminacies).

- Some easy examples of neutrosophic functions:

  f(3 or 5) = 9,

  i.e. not sure if f(3) = 9 or f(5) = 9;

- another example:

  g(4) = 3, 4, or 7,

  i.e. not sure if g(4) = 3, or g(4) = 4, or g(4) = 7.

- another one, more indeterminate:

  h(1 or 2) = 5 or 7.

But we also have thick (neutrosophic) functions. For example:

- see the graph at page 32;

- or better the graph at page 33.

Now, if we need e^(interval) for the interval neutrosophic complex set, for example

f(x) = e^([0.2, 0.4]·x), f: R → R, we graph the classical functions $f_1(x) = e^{\wedge}(0.2x)$ and $f_2(x) = e^{\wedge}0.4x$ and the <u>surface</u> between $f_1(x)$ and $f_2(x)$ will constitute f(x), that's why f(x) is named "thick".

If we compute, for example: f(2) = e^([0.2, 0.4]·2) = e^[0.4, 0.8] = [1.4918..., 2.2255...], then we see that f(2) is equal to a segment of line [from 1.4918..., to 2.2255...], not to a single point.

57 Hybrid Complex Neutrosophic Set

To Mumtaz Ali:

There can be possible in a Complex Neutrosophic Set to also consider the real parts of T, I, F being a special type of sets in [0, 1], while the imaginary parts of T, I, F being

another type of sets in [0, 1], especially if such things are need in applications.

I mean:

- the real parts be intervals, while the imaginary parts being single-valued (as you defined first time);
- or the real parts be hesitant sets, while imaginary parts single-valued.

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Mumtaz Ali:

I have an idea of a new representation of bipolar fuzzy set and the bipolar neutrosophic set. I mean complex fuzzy set and complex neutrosophic set can represent bipolar fuzzy set and bipolar neutrosophic set respectively. For example in complex fuzzy set, the amplitude term represent membership function which belongs to [1,0] while the phase term represent the negative counter values which belong [0, -1].

Similar case is for neutrosophic set.

What do you think?

Florentin Smarandache:

Yes, it can work. I explain the neutrosophic environment first (the fuzzy environment will be similar).

In general, in mathematical way, we may consider B = {the set of all bipolar neutrosophic sets} and C = {the set of all complex neutrosophic sets}.

Then we need to define an isomorphism:

$\alpha : B \dashrightarrow C$, $\alpha (x(T^+, I^+, F^+; T^-, I^-, F^-)) =$
$x((T^+)e^{\wedge}j(T^-), (I^+)e^{\wedge}j(I^-), (F^+)e^{\wedge}j(F^-))$, such that

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α (x*y) = α (x)* α (y) [neutrosophic relationship], where * is a neutrosophic operation used into the problem.

I.e. if one uses only the neutrosophic intersection, then * is the neutrosophic intersection.

If one uses both, the neutrosophic intersection and the neutrosophic union, then the above neutrosophic relationship should be valid for each of them.

We get thus a neutrosophic isomorphism.

*

Similarly for the intuitionistic fuzzy isomorphism between bipolar fuzzy set and complex fuzzy set. In general, in a mathematical way, we may consider B = {the set of all bipolar intuitionistic fuzzy sets}, and C = {the set of all complex intuitionistic fuzzy sets}.

Then we need to define an isomorphism
α : B -> C,
α (x(T^+, F^+; T^-, F^-)) = x((T^+)e^j(T^-), (F^+)e^j(F^-))
such that α (x*y) = α (x)* α (y) [intuitionistic fuzzy relationship], where * is a intuitionistic fuzzy operation used into the problem.

I.e. if one uses only the intuitionistic fuzzy intersection, then * is the intuitionistic fuzzy intersection. If one uses both, the intuitionistic fuzzy intersection and the intuitionistic fuzzy union, then the above intuitionistic fuzzy relationship should be valid for each of them. We get thus an intuitionistic fuzzy isomorphism.

And similarly for the fuzzy morphism.

58 DNA Quantum Computers

Florentin Smarandache to Christianto's Group:

Apropos of DNA Quantum Computer: I have a paper on Neutrosophic Quantum Computer, as an extension of classical Quantum Computer, but of course only theoretically...

It might be possible to extend it to DNA Quantum Computers.

59 Neutrosophic Approach of Error Rate Evaluation

Mehmet Serhat Can (Gaziosmapaşa Üniversity Zile Vocational School Tokat / Turkey):

I want to speed control of a direct current motor. Motor speed full range is between 0-6600 revolutions per minute. I want evaluate to speed range of the motor by using neutrosophic approach. Can I neutrosophication of speed range as 0-3200 interval is True value, 3100-3500 interval Indeterminate value, 3400-6600 interval is False value?

I am thinking a PID control strategy supported by the neutrosophicaiton for control of any system based on mentioned above.

Florentin Smarandache:

I see that the intervals of T and I overlap, and similarly the intervals of I and F overlap.

I defined in 2013 the Refined Neutrosophic Set and Logic (see my small paper from *Progress in Physics*).

My opinion is that you can refine as well the above application, i.e. 0-3100 is True value, then 3100-3200 is I_1 (subindeterminacy $I_1 = T \wedge I$), 3200-3400 is I_2 (subindeterminacy I_2), and 3400-3500 is I_3 (subindeterminacy $I_3 = I \wedge F$), and 3500-6600 is False value.

So you have T, I_1, I_2, I_3, F.

The neutrosophic operators are extended to refined neutrosophic operators.

You can refine it differently if you want. For example, using the same interval-values as for the first refinement:

T_1 = truth, T_2 = almost truth $(T \wedge I)$, I = indeterminacy, F_1 = almost false $(I \wedge F)$, and F_2 = false.

Now you have T_1, T_2, I, F_1, F_2.

Which refinement is better for you?

60 *Q-Single Valued Refined Neutrosophic Soft Sets*

To Qaisar Khan (Ph D student, Pakistan):

You can extend your paper: *Q-Single Valued Neutrosophic Soft Sets* to Q-Single Valued *Refined* Neutrosophic Soft Sets. Please see a small paper from 2013, the first paper where T, I, F were refined into: T_1, T_2, ... (types of subtruths); I_1, I_2, ... (types of subindeterminacies); and F_1, F_2, ... (types of subfalsehoods).

To Dr. Hong-yu Zhang (China):

You may be able to use in the development of the Neutrosophic Normal Cloud, the Neutrosophic Cognitive Maps and the Neutrosophic Relational Maps.

61 *Example of Dependent/Independent Neutrosophic Components*

If, for example, the degree of dependence $d°$ between *T* and *I* is *60%*, i.e. $d°(T, I) = 0.6$, then: $0 \le T + I \le 2 - 0.6 = 1.4$.

If the degree of dependence $d°$ between *I* and *F* is *20%*, i.e. $d°(I, F) = 0.2$, then: $0 \le I + F \le 2 - 0.2 = 1.8$.

If the degree of dependence $d°$ between *F* and *T* is, let'ssay, *0%*, i.e. $d°(F, T) = 0$, meaning that *F* and *T* are *100%* independent, then: $0 \le T + I \le 2 - 0 = 2$.

But because all *T, I, F* belong to the interval *[0, 1]*, one has: $0 \le T + I + F \le 2.4$; the maximum *2.4* occurs for *(T = 1, I = 0.4, F = 1)*, or for *(T = 0.7, I = 0.7, F = 1)* etc.

62 *Morphology & Neutrosophics*

Florentin Smarandache to Neutrosophic Group:

The Morphology (Study of Shapes) is the newest field that neutrosophics have just been applied, thanks to Prof. Dr. A. Salama from Port-Said University, Egypt.

63 *Neutrosophic Mathematical Morphology*

To E. M. El-Nakeeb, Hewayda ElGhawalby, A. A. Salama and S. A. El-Hafeez:

The paper on *Foundation for Neutrosophic Mathematical Morphology* is introduced for the first time, thanks to all authors. Just continue the good work in this field.

The neutrosophic operators, as the fuzzy operators, are approximations of aggregations.

There are many operators of the same type, I mean for each operation one has a class of operators.

For the neutrosophic negation, the most used one is: negation$(T, I, F) = (F, 1\text{-}I, T)$.

For the neutrosophic intersections and unions, the forms for I and F go together in the same sense.

$$(T_1, I_1, F_1) \wedge (T_2, I_2, F_2) = (T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2)$$
$$(T_1, I_1, F_1) \vee (T_2, I_2, F_2) = (T_1 \vee T_2, I_1 \wedge I_2, F_1 \wedge F_2)$$

For example,

- for neutrosophic dilation one can use:

  sup min / inf max / inf max

- and for neutrosophic erosion one can use:

  inf max / sup min / sup min.

In your paper is also okay, being a different approximation.

64 *Positive Weight of a Neutrosophic Set*

To Nouran Radwan:

H. Zhang, J. Wang, X. Chen, and Ridvan Sahin considered the neutrosophic sets as:

$A_1 \rightarrow (t_1, i_1, f_1)$'s for each x;

$A_2 \rightarrow (t_2, i_2, f_2)$'s for each element;

$A_3 \rightarrow (t_3, i_3, f_3)$'s for each element, etc.

Each neutrosophic set has a positive weight (importance): w_1, w_2, w_3, etc.

They compute:

- for the truth-value:

$[(1-t_1)^{\wedge}w_1][(1-t_2)^{\wedge}w_2][(1-t_3)^{\wedge}w_3]...$

- for indeterminacy-value:

$[i_1{}^{\wedge}w_1][i_2{}^{\wedge}w_2][i_3{}^{\wedge}w_3]...$

- similarly for the false-value:

$[f_1{}^{\wedge}w_1][f_2{}^{\wedge}w_2][f_3{}^{\wedge}w_3]...$

So, each neutrosophic set has a positive weight, as an importance; of course, the sum of weights is 1.

65 *Picture Fuzzy Set Refinement*

To Dr. Bui Cong Cuong and Dr. Pham Hong Phong (Vietnam):

You have introduced the picture fuzzy set in 2013, as particular case of single valued neutrosophic set, with $t + i + f = 1$, when all t, i, f are dependent.

What about studying the case when $t + i + f = 2$, i.e. when two components are dependent (for example t and i, then $t + i = 1$), and the third one f is independent from them.

Hence $t + I + f = 2$. This case has not been studied. We should also look for applications.

<div align="center">*</div>

To Dr. Bui Cong Cuong:

You can refine the picture fuzzy set, and get, let's say:

$$t_1 + t_2 + i_1 + i_2 + f_1 + f_2 = 1.$$

66 General Neutrosophic Complex Set

To Mumtaz Ali and Le Huang Son:

We can take, instead of "intervals", just any subsets from $[0, 1]$ for $t_{S(x)}$, $i_{S(x)}$, $f_{S(x)}$ and for $\varpi_{S(x)}$, $\theta_{S(x)}$, and $\nu_{S(x)}$.

Therefore, one has the General Neutrosophic Complex Set as:

- $T_{S(x)} = t_{S(x)}.e^{\wedge}(j \cdot \varpi_{S(x)})$,
- $I_{S(x)} = i_{S(x)}.e^{\wedge}(j \cdot \theta_{S(x)})$,
- $F_{S(x)} = f_{S(x)}.e^{\wedge}(j \cdot \nu_{S(x)})$,

where $j = \sqrt{-1}$.

67 (t,i,f)-Neutrosophic Algebraic Structures (I)

To Mumtaz Ali & Dr. Adesina Agboola:

What was never done in the field of algebraic structures is to consider the elements that only partially belong to the set. In classical algebraic structures an element belongs 100% to the set, but this is not always true in our everyday life, because for example in a factory there are workers who work part-time, so they only partially belong over-there. In an university there are faculty that also teach part-time, not full-time. And so on.

<div align="center">85</div>

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Then we define a law on the elements of the set, and another law on their neutrosophic components. For example:

Let's have a neutrosophic set

$A = \{1_{(0.1,0.4,0.6)}, 2_{(0.3,0.6,0.2)}, 3_{(1,0,0)}\}$, which means that the element "1" belongs only 10% to A, 40% its indeterminate appurtenance to A, and 60% it does not belong to A. Similarly for the elements 2 and 3.

Then we define a neutrosophic law in this way, for example:

$x(t_1,i_1,f_1)*y(t_2,i_2.f_2) = xy_{(\min\{t_1, t_2\}, \max\{i_1, i_2\}, \max\{f_1, f_2\})}$,

where "x·y" is x times y (as in arithmetic).

An example:

$$1_{(0.1, 0.4, 0.6)}*2_{(0.3, 0.6, 0.2)} =$$

$1\times2_{(\min\{0.1,0.3\}, \max\{0.4,0.6\}, \max\{0.6,0.2\})} = 2_{(0.1,0.6,0.6)}$.

Therefore, the law * is composed from a law L_1 between the elements x and y as in classical algebraic structures, and another law L_2 between the triplets (t_1, i_1, f_1) and (t_2, i_2, f_2) which are the neutrosophic components of the elements x and y respectively.

See into my book Simbolic Neutrosophic Theory the "(t, i, f)-neutrosophic algebraic structures". I only gave their definition, but they can be developed into various algebraic structures, like let's say: (t, i, f)-neutrosophic monoid, (t, i, f)-neutrosophic semigroup, etc.

I noted them as "(t, i, f)-neutrosophic structures" in order to distinguish them from the previous "neutrosophic structures" where one uses a+bI numbers (where I = literal

indeterminacy, and $I^2 = I$), with "a" and "b" real or complex numbers.

We may investigate if considering a law #:

$x_{(t_1,i_1,f_1)} \# y_{(t_2,i_2,f_2)} = \beta (x,y,t_1,i_1,f_1,t_2,i_2,f_2)$,

where β (........) is a function or operator of 8 variables.

I mean to mix the elements and their components...

Can that be possible in some applications?

68 (t,i,f)-Neutrosophic Algebraic Structures (II)

To Nouran M. Radwan, M. Badr Senousy and Alaa El Din M. Riad (Egypt):

Your paper Approaches for Managing Uncertainty in Learning Management Systems got to me too sent by the neutrosophic researcher and friend Dr. Ahmed Salama.

I think you can also use the neutrosophic probability in a future paper of yours on managing uncertainty, where the neutrosophic probability NP of an event E:

NP(E) = <chance that E occurs, indeterminate chance that E occurs, chance that E does not occur>.

An easy example:

There will be a soccer game between Egypt and Algeria. Then

NP(Egypt vs. Algeria) = <chance that Egypt will win, chance that there will be a tie game, chance that Egypt will loose>.

So, the neutrosophic probability is pretty easy and very much used in our everyday life.

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69 Neutrosophic Bipolar Graph

To Said Broumi:

Yes, we have a generalization of signed graph. I did the Neutrosophic Bipolar Graph, in a few ways:

- any vertex V_j has the neutrosophic bipolar value $(<T_j^+, T_j^->, <I_j^+, I_j^->, <F_j^+, F_j^->)$, where T_j^+, I_j^+, F_j^+ are the positive degrees of membership values in the interval $[0, 1]$, while T_j^-, I_j^-, F_j^- are the negative degrees of membership values in the interval $[-1, 0]$;

- similarly, any edge V_jV_k has the neutrosophic bipolar value $(<T_{jk}^+, T_{jk}^->, <I_{jk}^+, I_{jk}^->, <F_{jk}^+, F_{jk}^->)$, where $T_{jk}^+, I_{jk}^+, F_{jk}^+$ are the positive degrees of relationship values in the interval $[0, 1]$ between vertexes V_j and V_k, while $T_{jk}^-, I_{jk}^-, F_{jk}^-$ are the negative degrees of membership values in the interval $[-1, 0]$ between vertexes V_j and V_k.

I presented it to Vietnam in my past trip (together with our common paper on graphs).

For the signed graphs, the authors (Debanjan Banerjee, Anita Pal) took only the second part (i.e. the edges values) and only + or – signs, not numerical values.

Even we may consider a paper for IEEE Greece conference (up to 18 July 2016).

70 Neutrosophic Triplet Hedge Algebras

To Mumtaz Ali:

May we connect the hedge algebras with neutrosophic triplets, or extend it to neutrosophic triplet hedge algebras...

71 *Applications of Neutrosophic Triplet Sets*

To Dr. Adesina Agboola and Mumtaz Ali:

So far the applications of neutrosophic triplet sets are in Z, modulo n, n ≥ 2, you're right.

But I am thinking at new applications, for example in social science:

One person <A> that has an enemy <antiA$_{d_1}$> (enemy in a degree d_1 of enemycity), and a neutral person <neutA$_{d_1}$> with respect to <antiA$_{d_1}$>.

Then another enemy <antiA$_{d_2}$> in a different degree of enemycity d_2, and a neutral <neutA$_{d_2}$>, and so on.

Hence one has the neutrosophic triplets:

(<A>, <neutA$_{d_1}$>, <antiA$_{d_1}$>),

(<A>, <neutA$_{d_2}$>, <antiA$_{d_2}$>), and so on.

Then we take another person <B> in the same way, for example...

(<B>, <neutB$_{d_5}$>, <antiB$_{d_5}$>),

(<B>, <neutB$_{d_6}$>, <antiB$_{d_6}$>), etc.

I think we can get more applications, if we deeply think about cases where we have neutrosophic triplets (<A>, <neutA>, <antiA>) in technology and in science.

72 *Partial Algebraic Laws*

To Mumtaz Ali:

What about studying the partially defined algebraic laws?

Partial algebraic laws were never studied before.

Let $N^* = \{1, 2, 3, ...\}$, then the division "/" on $(N^*, /)$ is partially defined, since a/b belongs to N^* only if "a" is a multiple of "b".

To Mumtaz Ali:

Left Invertive Law: $(a*b)*c = (c*b)*a$, for all a, b, c in a groupoid S (Kazim and Naseeruddin, 1972), also called Abel-Grassmanní s groupoid (midway between a groupoid and a commutative semigroup).

But it is not the case for our neutrosophic triplets set on partially defined law *:

$(a*neut(a))*anti(a) = a*anti(a) = neut(a)$

and

$(anti(a)*neut(a))*a = anti(a)*a = neut(a)$

because we need to have $a*anti(a) = anti(a)*a$.

73 Neutrosophic Cognitive Maps

To Kanika Bhutani, Megha Kumar, Gaurav Garg, and Swati Aggarwal:

Your paper on Assessing IT Projects Success with Extended Fuzzy Cognitive Maps & Neutrosophic Cognitive Maps is interesting. It can be extended to (t,i,f)-Neutrosophic Cognitive Maps if you consider the (t,i,f)-neutrosophic graphs that Vasantha & I defined in 2003 (http://fs.gallup.unm.edu/NCMs.pdf), i.e. graphs whose nodes have the form (t,i,f), meaning the nodes belong to the graph with a membership "t", indeterminate membership "i" and nonmembership "f", and their edges

have also the form (t,i,f), meaning that the relationship between two nodes is (t,i,f).

So, it is possible to define a (t,i,f)-Neutrosophic Cognitive Map, which is an extension of the 2003 definition of NCM.

74 *Neutrosophic Overset / Overlogic / Overprobability*

To Mumtaz Ali:

I'll come back to you with the idea of neutrosophic overset/overlogic/overprobability because I tried to convince people long ago (2007), but they were very reluctant to it. I know you're very open mind, so might understand me. I had similar problems between 1995-1998 to convince people that the sum of components T+I+F can be up to 3, not necessarily up to 1.

75 *Singled Valued Neutrosophic Number Operations*

To Said Broumi:

Addition of Neutrosophic Numbers (which is actually like neutrosophic union):

$$(t_1, i_1, f_1) + (t_2, i_2, f_2) = (t_1 + t_2 - t_1 t_2, i_1 i_2, f_1 f_2).$$

Multiplication of Neutrosophic Numbers (which is actually like neutrosophic intersection):

$$(t_1, i_1, f_1) \times (t_2, i_2, f_2) = (t_1 t_2, i_1 + i_2 - i_1 i_2, f_1 + f_2 - f_1 f_2).$$

For the Subtraction of Neutrosophic Numbers:

$$(t_1, i_1, f_1) - (t_2, i_2, f_2) = (\ (t_1 - t_2)/(1 - t_2)\ , i_1/i_2, f_1/f_2),$$

of course for t_2 different from 1, and i_2, f_2 different from 0; also $t_1 \geq t_2$ (otherwise 0) and $i_1 \leq i_2$ (otherwise 1) and $f_1 \leq f_2$ (otherwise 1).

For the Division of Neutrosophic Numbers:

$(t_1, i_1, f_1) / (t_2, i_2, f_2) = (t_1/t_2, (i_1-i_2)/(1-i_2), (f_1-f_2)/(1-f_2))$, of course for t_2 different from 0, and i_2, f_2 different from 1; also $t_1 \leq t_2$ (otherwise 1) and $i_1 \geq i_2$ (otherwise 0), $f_1 \geq f_2$ (otherwise 0).

We can then straightforwardly generalize them to interval-valued neutrosophic number additions and divisions.

76 Neutrosophic Label Graph

To Said Broumi:

Let $L = \{L_1, L_2, ..., L_n\}$, $n \geq 2$, a set of labels (for example: extremely bad, very bad, bad, medium, a little good, good, very good etc.).

For each element x, from the universe of discourse U, one has $x(T(x), I(x), F(x))$, where T, I, F: U $--------$> L.

Hence each vertex V_j has the neutrosophic label values $V_j(T_j(x), I_j(x), F_j(x))$ and each edge V_jV_k has the neutrosophic label values $V_{jk}(T_{jk}(x), I_{jk}(x), F_{jk}(x))$.

This is a neutrosophic label graph.

For example:

$$(\{L_6\}, \{L_1, L_2\}, \{L_3\})$$

$A(\{L_2, L_3, L_4\}, \{L_1\}, \{L_4, L_5\})$ ----------------------> $B(\{L_4, L_6\}, \{L_2\}, \{L_5, L_7\})$.

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77 Neutrosophic Triangular, Trapezoidal, Bell Shapes

To Sisalah Bouzina:

$T_A(x)$ can be as in Fuzzy Set: triangular, trapezoidal, bell shape, etc.

While both $I_A(x)$ and $F_A(x)$ are also triangular (with the vertex down), trapezoidal (with the smaller base down), or bell shape (the bell is downward), etc. respectively, i.e. $I_A(x)$ and $F_A(x)$ are upside-down/opposite with respect to $T_A(x)$.

78 Neutrosophic Linear Goal Programming

To Surapati Pramanik:

I read yesterday your interesting paper on Neutrosophic Linear Goal Programming. Thanks for the history of neutrosophics... Indeed, the beginning is not easy.

By the way, you may try to extend your paper from neutrosophic set to neutrosophic overset/underset/offset. Never in the history of science was it considered the membership degree > 1 or < 0, but now, eventually these new ideas are accepted by the mainstream.

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To Prof. Huda E. Khalid, Ahmed K. Essa:

In order to advance the research in neutrosophic GP we need to define: a function $f(x)$ is neutrosophically less than or equal to another function $g(x)$.

79 Fonctions Continues de Croyances

À Jean Dezert:

J'ai parlé à Speyer avec le Prof. Jean-Philippe Lauffenburger, de l'Université de Haute-Alsace.

Il avait un étudient doctorant (Jeremy) que j'ai rencontré à Singapore à la Fusion aussi.

Jeremy travaillait sur les fonctions de croyances continues...

Qu'est-ce que tu penses des fonctions continues de croyances?

80 Spheroidal Neutrosophic Set

To Mumtaz Ali:

I remember we tried to do something about the spheroidal neutrosophic set, where: $T^2 + I^2 + F^2 = 3$, which means that *T, I, F* are in $[0, \sqrt{3}]$, with $\sqrt{3}] = 1.73... > 1$.

Yes, it can work in the neutrosophic overset, where the degree of membership / indeterminacy / nonmembership is allowed to be over 1.

We may have overmembership.

For example, a worker W_1 that works full-time has the degree of membership 1, but another worker W_2 that works over-time has the degree of membership with respect to his company over 1 (> 1).

We may even extend further to:

$- \sqrt{3}] \le T, I, F \le \sqrt{3}]$,

and we get a cube with a neutrosophic offset (i.e. overmembership and undermembership).

We need an example, from technique, from science, from commerce, from social science when the maximum allowed overtime is $\sqrt{3}$].

If $T^2 + I^2 + F^2 = 3$ and $T, I, F \in [0, \sqrt{3}]$,
then one has only $1/8$ of the sphere of center $O(0,0,0)$ and radius $\sqrt{3}$].

If $T^2 + I^2 + F^2 = 3$ and $T, I, F \in [-\sqrt{3}], \sqrt{3}]]$,
then one has the whole sphere of center $O(0,0,0)$ and radius $\sqrt{3}$].

81 Multipolar Neutrosophic Soft Sets

To Mumtaz Ali:

We need to do the extension from Bipolar Neutrosophic Soft Sets to Tripolar (and Multipolar) Neutrosophic SOFT Sets.

82 Neutrosophic Overset, Neutrosophic Underset, Neutrosophic Offset

To Chunfang Liu:

The neutrosophic set is extended to neutrosophic overset (when there are elements whose membership degree is > 1, for example in a company where some people work overtime), to neutrosophic underset (when there are elements whose membership degree is < 0, for example in a company where some people produce more damage than profit so these people have a negative membership), and neutrosophic offset (when there are elements with

membership degree > 1 and elements with membership degree < 0).

83 Spheroidal Neutrosophic Overset, Underset, Offset

To Mumtaz Ali:

If we consider $0 \leq T, I, F \leq \sqrt{3}$, i.e. neutrosophic components' values > 1 only, then we have *spheroidal neutrosophic overset*.

If we consider $-\sqrt{3} \leq T, I, F \leq 1$, i.e. neutrosophic components' values < 0 only, then we have *spheroidal neutrosophic underset*.

If we consider $-\sqrt{3} \leq T, I, F \leq \sqrt{3}$, i.e. neutrosophic components' values > 1 and < 0, then we have *spheroidal neutrosophic offset*.

84 Neutrosophic Graphs

To Said Broumi:

One can extend the bipolar neutrosophic graph to a Tripolar Neutrosophic Graph, i.e. with T^+ (positive), T^o (neutral), T^- (negative); and similarly: I^+, I^o, I^-; and F^+, F^o, F^-.

And a neutrosophic graph to a *Neutrosophic Offgraph*, i.e. let's consider a company as a neutrosophic offgraph, where each vertex is an employee of this company:

- some employees work overtime, hence their membership degree > 1;

- others work just full-time, hence their membership degree = 1;

- others work part-time, hence their membership degree is in (0, 1);

- others are absent all time, hence their membership degree = 0;

- and the last ones do not work at all and produce much damage to the company,

hence their membership degree < 0.

To Said Broumi:

I saw that that in fuzzy graphs and intuitionistic fuzzy graphs there is the restriction that

$T(v_1, v_1) \leq \min\{T(v_1), T(v_2)\}$.

I think we do not need any restriction, since if v_1 and v_2 belong to the graph each of them in a certain degree of membership, that does not mean that the relationship between v_1 and v_2 has to have a degree of membership affected by the memberships of v_1 and v_2 to the graph.

85 Neutrosophic Triplets

Florentin Smarandache:

I think we can define neutrosophic triplets in R_n, i.e. set of real numbers modulo n (not only in Z_n, i.e set of integers modulo n).

We can consider the law $a*b = 2a + 2b$ on R.

Then the neutrosophic triplets are: $(a, -a/2, -5a/4)$, for any a in R.

86 Neutrosophic Triplet Sets

To Mumtaz Ali:

We need to make distinction between neutrosophic triplet sets, where one has <a, neut(a), anti(a)>, and the duplets of the form <a, neut(a)> or the form <a, anti(a)>.

Let's remain consistent and consider a neutrosophic triplet ring/field/vector_space (M, *, #) such that that both (M,*) and (M, #) are neutrosophic triplet sets, satisfying each of them some axioms.

We may then define another category of structures: where (M, *) and (M, #) may be: one a set of neutrosophic triplets, another a set of neutrosophic duplets <a, neut(a)> or <a, anti(a)>.

Or both of them neutrosophic duplets.

We need to give another name to these new structures.

87 Neutrosophic Triplet Rings, Neutrosophic Triplet Fields, Neutrosophic Triplet Vector Spaces

To Mumtaz Ali:

We need to extend the Neutrosophic Triplet Structures from one law *, to two laws * and #, and with respect to each law we need to have triplets
<a, neut*(a), anti*(a)>, which is a neutrosophic triplet with respect to the law *, and respectively
<a, neut#(a), anti#(a)>, which is a neutrosophic triplet with respect to the law # {of course, these two triplets are

in general different from each other}, in order to define the Neutrosophic Triplet Rings, the Neutrosophic Triplet Fields, and to further extend to three laws in order to define the Neutrosophic Triplet Vector Spaces etc. Many applications of them we have to present.

88 Degrees of Membership

To Dmitri Rabounski:

I have extended my neutrosophic logic / set / probability to the case when the degree of membership of an element is > 1 (for ex. somebody working overtime), and when the degree of membership of an element is < 0 (for example a spying double-agent that brings damage to his country). It was never done before in the history of science and I expected that people will start to denigrate and insult me, yet because of simple examples I gave, it was accepted by the mainstream in *arXiv.org*...

89 Neutrosophic Triplet Group, Neutrosophic Duplet Algebraic Structures

Florentin Smarandache:

We should consider a *neutrosophic triplet algebraic structure* (M, *, #), if both (M, *) and (M, #) are neutrosophic triplet sets.

Mumtaz Ali:

Yes, and Neutrosophic Triplet Group (NTG) is the first paper in this area.

Florentin Smarandache:

NTG is for one law only (*). We need a paper for NT structure with two laws (* and #).

Florentin Smarandache:

Now, we may consider *neutrosophic duplet algebraic structure*s (M, *, #), if both (M, *) and (M, #) are neutrosophic duplet sets, i.e. of the form <a, neut(a)>.

We should define the neutrosophic duplets as:

For a set (M,*) we have a neutro-duplet if for each "a in M" there is a "neut(a) in M" such that

a*neut(a) = neut(a)*a, with neut(a) different from the unity element, and there is no "anti(a)" such that

a*anti(a) = anti(a)*a = neut(a).

Mumtaz Ali:

Yes, neutrosophic duplet algebraic structures such as neutrosophic duplet monoid, neutrosophic duplet LA-semigroup (LA monoid) etc.

Florentin Smarandache:

And, we should call *neutrosophic duplet-triplet algebraic structures* (M, *, #), if one of (M, *) and (M, #) is a neutrosophic triplet set and the other one is a neutrosophic duplet set.

Mumtaz Ali:

The combination of these two gives us neutrosophic duplet-triplet ring in the classical ring structure.

Florentin Smarandache:

In many aggregations of neutrosophic sets, the second component (I) is computed in the same way as the third component (F).

Surely, there may be exceptions, depending in the expert.

90 Total Order of Neutrosophic Numbers

To Surapati Pramanik:

One total order for neutrosophic numbers of the form (t, i, f), can be: $(t_1, i_1, f_1) > (t_2, i_2, f_2)$ if:

$t_1 > t_2$;

or $t_1 = t_2$, but $i_1 < i_2$;

or $t_1 = t_2$, $i_1 = i_2$, but $f_1 < f_2$.

To Said Broumi:

Let $N_1 = a_1 + b_1 I$, $N_2 = a_2 + b_2 I$ be two neutrosophic numbers, formed by determinate part + indeterminate part.

Then $N_1 - N_2 = (a_1 + b_1 I) - (a_2 + b_2 I) =$

$(a_1 - a_2) + (b_1 - b_2)I$.

91 Neutrosophy is a Generalization of Paradoxism

To Surapati Pramanik:

Neutrosophy is a generalization of paradoxism (see http://fs.gallup.unm.edu/a/paradoxism.htm).

The paradoxism is based on contradictions (<A> and <antiA>), antinomies, paradoxes, oxymorons etc.

I started the paradoxism in 1980 in literature, arts, science.

Neutrosophy extended the paradoxism by introducing the <neutA> in between <A> and <antiA>, so neutrosophy uses all three: <A>, <neutA>, and <antiA>.

92 *Definition of Type-2 (and Type-n) Neutrosophic Set*

Type-2 Neutrosophic Set is actually a neutrosophic set of a neutrosophic set.

See an example for a type-2 single-valued neutrosophic set below:

Let x(0.4 <0.3, 0.2, 0.4>, 0.1 <0.0, 0.3, 0.8>, 0.7 <0.5, 0.2, 0.2>) be an element in the neutrosophic set A, which means the following: x(0.4, 0.1, 0.7) belongs to the neutrosophic set A in the following way, the truth value of x is 0.4, the indeterminacy value of x is 0.1, and the falsity value of x is 0.7 [this is type-1 neutrosophic set]; but the neutrosophic probability that the truth value of x is 0.4 with respect to the neutrosophic set A is <0.3, 0.2, 0.4>, the neutrosophic probability that the indeterminacy value of x is 0.1 with respect to the neutrosophic set A is <0.0, 0.3, 0.8>, and the neutrosophic probability that the falsity value of x is 0.7 with respect to the neutrosophic set A is <0.5, 0.2, 0.2> [now this is type-2 neutrosophic set].

So, in a type-2 neutrosophic set, when an element x(t, i, f) belongs to a neutrosophic set A, we are not sure about the values of t, i, f, we only get them with a given neutrosophic probability.

Neutrosophic Probability (NP) of an event E is defined as: NP(E) = (chance that E occurs, indeterminate chance about E occurrence, chance that E does not occur).

Similarly, a type-2 fuzzy set is a fuzzy set of a fuzzy set. And a type-2 intuitionistic fuzzy set is an intuitionistic fuzzy set of an intuitionistic fuzzy set.

Surely, one can define a type-3 neutrosophic set (which is a neutrosophic set of a neutrosophic set of a neutrosophic set), and so on (*type-n neutrosophic set*, for n ≥ 2), but they become useless and confusing.

Neither in fuzzy set nor in intuitionistic fuzzy set the researchers went further that type-2.

93 Neutrosophic Graph Applied in Medicine

We may consider the graph vertices as disease symptoms.

Then a graph walk through, for example, vertices V_1, V_3, V_7, ... etc. can lead to disease d_1. And so on.

Of course, we consider neutrosophic degrees of each vertex, of each edge, and get a neutrosophic degree for each disease.

94 *Neutrosophic Multiplicative Set*

To Ridvan Sahin:

To be similarly with the Intuitionistic Fuzzy Multiplicative Set (IFMS), we may consider within the environment of Neutrosophic Multiplicative Set (NMS): $1/9 \leq T, F \leq 9$, and $1/9 \leq I \leq 9$, where $0 \leq T \cdot F \leq 1$, so $0 \leq T \cdot I \cdot F \leq 9$, which is actually equivalent to your: $0 \leq (T \cdot I \cdot F)^{1/2} \leq 3$.

By the way, not quite related to NMS, I have considered the case when the degrees of membership/indeterminacy/nonmembership can be > 1 or < 0, I mean T, I, F can be bigger than 1 and less than 0 in certain cases.

Can we connect NMS with neutrosophic overset/underset/offset? By the way, do you have in mind any possible application of NMS?

Another idea for the NMS. In 2013 I have extended the neutrosophic set to refined neutrosophic set, i.e. T, I, F refined into $T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...$.

The best way will be to consider the Refined Neutrosophic Multiplicative Set, in the following way: $T, I_1, I_2, F,$
where T = preferred degree,
we refined only "I" into
I_1 = the indeterminate preferred degree,
and I_2 = the indeterminate nonpreferred degree,
and F = nonpreferred degree,

with $1/9 \leq T, F \leq 9$ and $0 \leq T \cdot F \leq 1$,

and $1/9 \leq I_1, I_2 \leq 9$ and $0 \leq I_1 \cdot I_2 \leq 1$.

The operators on refined neutrosophic sets are, in general, extensions of the operators on neutrosophic set.

Examples:

i) if a, b are neutrosophic sets, then

a·b = (a · b, a+b-a·b, a+b-a·b);

ii) but if a, b are refined neutrosophic sets (refined as T, I_1, I_2, F as we need in RNMS), then similarly:

a·b = (a·b, a+b-a·b, a+b-a·b, a+b-a·b).

I mean, the subcomponents I_1 and I_2 from the RNMS behave similarly to the component "I" from NMS.

Then:

iii) if a, b are neutrosophic sets, then:

a+b = (a+b-a·b, a·b, a·b);

iv) but if a, b are refined neutrosophic sets, similarly

a+b = (a+b-a·b, a·b, a·b, a·b).

Etc.

We can consider in NMS, as most of the times in the neutrosophic set, that indeterminacy "I" is independent from T and F. But T and F are kind of dependent in NMS, since $T \cdot F \leq 1$.

[Please see the paper: *Florentin Smarandache, Degree of Dependence and Independence of the (Sub)Components of Fuzzy Set and Neutrosophic Set, Neutrosophic Sets and Systems, vol. 11, 2016, pp. 95-97,* http://fs.gallup.unm.edu/NSS/DegreeOfDependenceAndIndependence.pdf, about dependence and independence between T, I, F, i.e.

if T, I, F are all dependent then T + I + F = 1,

if T, I, F are independent two by two, then T + I + F = 3,

if two components are dependent, while the third one independent from them, then T + I + F = 2,

etc.

We may also have "degrees of dependence and independence between components"; for example T and F can be dependent only in a percentage of let's say 60%, etc.

The attached paper may inspire you to write new papers, since besides the attached paper no other work/research was published on dependence / independence of components.

Coming back to our NMS: yes, we can consider $1/9 \leq I \leq 9$ and "I" as independent from T and F.

95 Neutrosophic Filter

To Arsham Borumand Saeid & Akbar Rezaeii:

A neutrosophic filter is a set A, which in the classical way [when each element belongs 100% to the set, i.e. x(1, 0, 0)] satisfies the definition of an implicative filter, while when the elements x have neutrosophic degrees of appurtenance to the set A, i.e. $x(t_A, i_A, f_A) \in A$, then more axioms have to be satisfied with respect to the neutrosophic components (t_A, i_A, f_A).

96 *Refined Neutrosophic Logic*

To Prof. Prem Kumar Singh:

Neutrosophic logic is more than a tripolar logic (as discussed by Kleen, Lukasiewicz, Pawlak, and Yao), as you say, it is a multi-polar infinite logic, where the truth (T), indeterminacy (I), and falsehood (F) have been split into respectively:

$T_1, T_2, ...$ (sub-truths),

$I_1, I_2, ...$ (sub-indeterminacies),

$F_1, F_2, ...$ (sub-falsehoods),

called refined neutrosophic logic.

The Yin Yang logic of China, which is similar to the Dialectics by Hegel, Marx and other philosophers, were based on the opposites, let's denote them by <A> and <antiA> (where <antiA> is the opposite of <A>). Similarly, for the Sanskrit language. But they were extended to Neutrosophy, where to the opposites <A> and <antiA> one adds the neutralities between them (let's denote them by <neutA>). This is because in our real life, for example if there is a conflict between two opposites (let's say between two countries, India and Pakistan), some neutral countries also interfere, by helping or boycotting either India or Pakistan. Therefore, Neutrosophy is more than Yin-Yang and more than Dialectics.

By the way, Lukasiewicz's three valued logic is finite (truth, unknown, falsehood are singled valued), while neutrosophic logic is triple-infinite (T has an infinite number of values in [0, 1]; similarly I and F).

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Even more, the neutrosophic logic was extended to neutrosophic offlogic (see the file, accepted by the mainstream, where the truthvalues > 1 or < 0 were allowed deepening on the applications:
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf).

The refinement of the truthvalues and their extension outside the interval [0, 1] were not before in logics.

97 *Triangular Neutrosophic Numbers / Intervals*

To Sisalah Bouzina:

1) In the case when $T(x)$, $I(x)$, $F(x)$ are single-valued numbers in [0, 1].

$T(x)$ is: first part an elementary equation of a line that increases; and the second an elementary equation of a line that decreases.

The two segments of line (up and down) should be framed/bounded inside [0, 1].
Similarly, $I(x)$ and $F(x)$ are formed by segments of line that are included into [0, 1].

2) In the case when $T(x)$, $I(x)$, $F(x)$ are intervals included in [0, 1].
For the increasing lines: You draw a line that represents the "inf" of $T(x)$ and another line that represents the "sup" of $T(x)$. Then you take the surface in between the two lines. For the decreasing lines you do the same.

Similarly for $I(x)$ and $F(x)$.

98 *Big Data*

Au Prof. Arnaud Martin:

Est-ce qu'il y a des règles spéciales pour la *big data* ?

Est-ce l'on peut adapter les PCRs et autres règles antérieures dans la fusion de la *big data* ?

Quelle sera la plus simple règle pour combiner beaucoup de masses avec une grande cadre ? Faire la moyenne arithmétique ?

Ou simplifier / réduire la complexité des masses ?

99 *Neutrosophic Precalculus and Neutrosophic Calculus*

To Hoda Esmail:

Besides my book on neutrosophic precalculus and neutrosophic calculus, no more study has been done.

You're the first to try doing so. If you look at my volume you translated to Arabic, the classical theorems from precalculus / calculus do not exactly transpose into the neutrosophic precalculus / calculus. You observed, for example, that besides: left limit, right limit, and limit, in neutrosophic calculus we also have neutrosophic mereo-limit (king of partial-limit). If you study the L'Hospital Rule, and you'll find out that, because of the indeterminacy inserted into the neutrosophic functions, one has a Neutrosophic L'Hospital Rule, i.e. partially valid, partially indeterminate and partially invalid classical L'Hospital Rule. We can try various examples. Similarly for other undefined operations in classical calculus.

100 Real and Imaginary Indeterminacy

To Santanu Kumar Patro:

We may split the indeterminacy into "real indeterminacy" and "imaginary indeterminacy".

Try to get more convincing examples and practical applications, then this will be proved by them.

101 Multiple-Criteria Methods

Florentin Smarandache:

Interesting article "Is the evaluation of the students' values possible? An integrated approach to determining the weights of students' personal goals using multiple-criteria methods", by Stanislav Dadelo, Zenonas Turskis, Edmundas Kazimieras Zavadskas & Tomas Kačerauskas (ResearchGate.net, October 2016)

My answer to your question is neutrosophic: t% possible, i% indeterminate (because of some hidden parameters), and f% impossible.

Now the problem/research is: to find *t, i, f.*

102 Neutrosophic Triplet Structures

We may consider maybe two types of neutrosophic triplet structures, endowed with two internal laws * and #:

a) when the neutrosophic triplets with respect to * are the same as the neutrosophic triplets with respect to #;

b) when the neutrosophic triplets of * are different from those with respect to #.

103 Refined Residual Offlattice

To Prof. Prem Kumar Singh:

Going deeper into your paper on "Three-way fuzzy concept lattice representation using neutrosophic set" (Int. J. Mach. Learn. & Cyber., 2016), I think it would be possible for you to extend the acceptation/rejection/uncertainty to refined acceptation / refined rejection / refined uncertainty. See attached a such paper, where T, I, F are refined into T_1, T_2, ...; I_1, I_2, ...; F_1, F_2, ...

Since you also talked about super-concept and sub-concept, the second file might give you the idea of considering degree of truth-membership > 1 for super-concept, and degree of truth-membership < 0 for sun-concept.

I presented elementary examples in the second book of cases when may have in our everyday life degrees of truth-membership > 1 or < 0.

The Residual Lattice can also be extended to include values off the interval [0, 1], as a residual offlatice.

104 Nedeterminare

Către Dr. Gheorghe Savoiu, Universitatea din Pitești:

Consider că orice om are un procent de stupiditate / imbecilitate / incompetență (s%), un procent de naivitate (n%), un procent de inteligență (t%), și un procent de preluare/jefuire (comasând Legile lui Cipolla) -- depinzând și de domeniul la care este referit.

Mulțumesc pentru citarea *Legilor lui Florentin*; aveți dreptate, ele sunt între extreme (Legile lui Murphy și Legile lui Peter), ca nedeterminarea în Logica Neutrosofică.

105 Neutrosophic Permittivity / Tunneling / Coherent Correlated States

To Vic Christianto & Volodymyr Krasnoholovets:

I read the papers you sent me, especially about LENR (low energy nuclear reactions). They gave me the ideas of extensions to neutrosophic set and logic (as you have suggested). For examples:

- neutrosophic permittivity, which means a degree (t_P) of permittivity, a degree (i_P) of indeterminate-permittivity, and degree (f_P) of nonpermittivity; which is more realistic, because of much indeterminacy/neutrality in between opposites in our world;

- similarly we have neutrosophic tunneling;

- and between interacting particles, one has a degree of CCS (coherent correlated states), degree of indeterminate-coherent correlated states (ICCS), and degree of noncoherent correlated states (NCCS).

How can we use these into the Schrodinger-Robertson uncertain relation?

By the way, the uncertainty can be refined into types of uncertainty (unknown, contradictory, incomplete etc.). Then, can we refine the Schrodinger-Robertson uncertain relation?

106 *Spheroid Neutrosophic Set*

To Mumtaz Ali:

We generalize the Pythagorean Intuitionistic Fuzzy Set, where $T^2 + F^2 \leq 1$, to Spheroid Neutrosophic Set (also called Generalized Pythagorean Neutrosophic Set, or just Pythagorean Neutrosophic Set) - since saying "neutrosophic" one understands 3 components whose maximum sum is 3, and Pythagorean means the sum of their squares.

Hence in our neutrosophic case we have:

$T^2 + I^2 + F^2 \leq 3$, with sphere radius $\sqrt{3}$.

Since all $0 \leq T, I, F \leq 1$, our neutrosophic spheroid is 1/8 of the sphere $T^2 + I^2 + F^2 = 3$, similarly to the Pythagorean Intuitionistic Fuzzy Set which is ¼ of the circle $T^2 + F^2 = 1$.

107 *Neutrosophic Finite State Machines*

I think we can extend Zadeh's inequality in a fuzzy relationship, and we can consider the vertexes' and the edges' neutrosophic values as independent values in a neutrosophic graph (similarly as T, I, F are independent components in neutrosophics, but not independent in fuzzy theory). I mean we can remove Zadeh's inequality in a fuzzy relationship and in a neutrosophic relationship.

For example, in a neutrosophic graph, the vertexes' values (t_1, i_1, f_1) and edges' values (t_2, i_2, f_2) are not restricted in any way.

Example:

$$(0.6, 0.1, 0.2)$$

A(0.5, 0.4, 0.3) ---------------------------------- B(0.1, 0.3, 0.2)

Hence,

0.6 = T(A,B) > min{T(A), T(B)} = min{0.5, 0.1} = 0.1, so Zadeh's inequality is not needed since the sources can be independent.

We can also extend your work on neutrosophic graphs to Neutrosophic Finite State Machines.

108 Multifractal Set Theory & Neutrosophics

Dr. Ervin Goldfain (Syracuse, NY, USA):

Am găsit recent o serie de rezultate extrem de promiţătoare (ataşez articolul).

În particular, folosind "*multifractal set theory*" (care are aplicaţii importante în teoria sistemelor neliniare şi a chaosului), am reuşit să arăt că dimensiunea fractală a Relativităţii Generale coincide cu cele patru dimensiuni ale spaţiu-timpului clasic (D = 4), pe când dimensiunea fractală a lui *Standard Model* este D = 2. Aceasta dimensiune (D = 2) este exact dimensiunea mişcării Browniene (*Brownian motion*) şi, totodată, dimensiunea tuturor "*quantum paths*" care descriu evoluţia în spaţiu-timp a obiectelor cuantice (inclusiv particulele elementare).

Este relativ "*straightforward*" să se demonstreze că orice dimensiune fractală intermediară între D = 2 şi D = 4 marchează existenţa unui domeniu care nu este nici "cuantic" (*Standard Model*) nici clasic (*General Relativity* în

patru dimensiuni). Adică, o stare neutrosofică - care corespunde lui "*Dark Sector*" (*dark matter* și *dark energy*).

Florentin Smarandache:

Desigur, precum nemateria, în general partea de mijloc, dintre opoziții, este echivalentul lui <neutA> dintre <A> și <antiA>.

109 *Questions on Neutrosophy*

Santanu Kumar Patro (India):

1. Can this Neutrosophic Statistics applied to Industrial Management study?

2. Can we apply it with the study of Digital Signal Processing?

3. Can we merge the Representation theory with Neutrosophy for a new theory?

4. Is the uncertain theory, K-theory solve the recent intriguing statistical problems by the power of this Neutrosophic Logic?

5. Can we construct a special master-space by the fusion of manifold concepts, soft topology, Ergodic theory, with Neutrosophic Distribution?

6. Is it possible for the construction of Neutrosophic Manifold?

7. Is it possible for the construction of Neutrosophic Algebraic Geometry?

Florentin Smarandache:

The answer to your last questions is: YES. In each field where there is so indeterminacy / contradiction / conflict one can use the neutrosophic logic and its fields: neutrosophic set, neutrosophic measure, neutrosophic integral, neutrosophic dynamic system, neutrosophic probability, neutrosophic statistics, neutrosophic algebraic structures, etc.

Nguyễn Văn Minh:

I am interested in the *Neutrosophic Bayesian Statistics*.

110 Neutrosophic Theory

A. A. A. Agboola:

I hope we are not doing badly in neutrosophic algebraic structures. It will take some time for some conservative mathematicians to embrace this new concept and some may not even embrace it at all as it happens to the concept of fuzzy sets. One of my collaborators Prof Bijan Davaz, a well known mathematician (algebraist), is still doubting...

Florentin Smarandache:

Surely, there may be people who do not understand the fuzzy set and neutrosophic set come from our every day.

Let's say a student John at your institution takes only one course, not 5 as required for a full-time student.

Therefore, John's membership (appurtenance) to your university is only 1/5 = 0.2. Another student George may take 2 courses only, so George's membership is 2/5 = 0.4. And so on.

For a future proposition, you cannot have a 100% truth or falsehood percentage; for example you cannot know exactly if in 2017 there will be a terrorist attack in US.

You can only approximate, so you have to use fuzzy logic or neutrosophic logic.

I do not know why you're so obsessed with that professor that is not very well known...

Ask him how does he deals with not well known elements, or not well known propositions that we need to measure their truth values? Can he use the Boolean logic to measure the proposition P = "In 2017 there will be a terrorist attack"? Because in Boolean logic he has to say either P = 0 or P = 1 [only God can say this!].

There are things that are neither black nor white, but also gray...

Huda Esmail:

It does not harm the sun if someone used a sieve to hide it, so what, if some few peoples don't understand or don't accept realities like fuzzy logic or neutrosophic logic?

Florentin Smarandache:

Yes, it is surprising that after decades of research and many indisputable applications of fuzzy theory even of

neutrosophic theory by hundreds of scientists around the world, there are still people miss-confident in them.

In 2003, when I presented the neutrosophic set and logic at the University of Berkeley, in California, I met Prof. L. Zadeh in person. Prof. Zadeh appreciated the neutrosophic set and logic and encouraged me to continue.

Also, he confessed to me that he faced much opposition, critics and even humiliation from conservative mathematicians of his time (1965, when he started the revolutionary FUZZY SET).

111 Neutrosophication of Lao Tse's Ideas

To Fu Yuhua:

I think we can consider the neutrosophication of Lao Tse's ideas, or of another branch of Chinese philosophy.

112 Neutrosophy in Indian Philosophy

To Dr. Surapati Pramanik:

One mixes "rough" with "neutrosophic", which was only very little treated. Another possible cooperation using neutrosophy. See attached a sample from neutrosophy in Chinese philosophy (about Taoism).

Therefore, we can insert neutrosophy in Indian philosophy (of course, a school only from Indian philosophy), in the following way: we combine <A>, <neutA>, <antiA> together. I mean <A> = an Indian philosophical school of ideas; then we design/define the <antiA> (the opposite ideas), and also <neutA> (the

neutral ideas in between the opposites - like in neutrosophics).

And get new ideas, which are neutrosophic in nature.

Or a philosopher from your university may cooperate with us.

To Sarfaraz Hashemkhani Zolfani, Edmundas Kazimieras Zavadskas, Zenonas Turskis:

Thank you for your Yin-Yang paper (dialectics from China): *Design of Products with Both International and Local Perspectives based on Yin-Yang Balance Theory and Swara Method* (2015).

I have worked three months in Guangzhou.

Besides dialectics, and as a generalization of dialectics (based on dynamic of opposites)

I considered the neutrosophy (based on dynamic of opposites and of the neutrality in between them - since in our world, the neutral <neutA> between opposites <A> and <antiA> interfere and help one side or the other; for example, two countries fighting in a war; the neutral countries, some of them, interfere and help one country or the belligerent country).

From neutrosophy, I went to neutrosophic set, neutrosophic logic, neutrosophic probability, etc.

113 *Comments on Neutrosophy*

I had hard time between 1995-1998 when I tried to publish papers on neutrosophic logic, set or probability. I

was humiliated that I didn't know elementary things about "probability", because the sum of distance space probabilities had to be 1, not 3 as in neutrosophic logic, set and probability.

I knew I'll face opposition and misunderstanding. People had the brain washed by traditional (objective) probability, yet subjective probability is different. If the three neutrosophic componets are independent and they privide information about respectively truth /indeterminacy falsehood of a proposition, or membership / indeterminacy / nonmembership of an element with respect to a set, or chance/ indeterminacy / nonchance of an event to occur then the sun of these three may get up to 3.

When at least one of the neutrosophic components, T, I, F is over 1, i.e., strictly greater than 1, or strictly below zero, one has a neutrosophic offset.

This has never been done before in the fuzzy set, fuzzy logic, intuitionistic fuzzy set, intuitionistic fuzzy logic, classical probability, or imprecise probability.

Such idea that a neutrosophic component may be strictly greater than 1, depending on the application we work on, comes actually from our every day life.

It may look shocking with respect to the classical set theory, classical logic, classical measure theory and so on, but it is perfectly normal to have a degree greater than 100%! But there are such cases in our every day life.

Let's see some examples.

In this example, we need to make a distinction between the students who enroll in less than 15 credit

hours (part-time students), students who enroll in 15 credit hours (full-time students), and students who enroll in more than 15 credit hours (over-full-time students) from administratively and financially poits of views.

Answering Dr. Surapati Pramanik's Questions.
Florentin Smarandache:

Your daily reading time = all the time (except when I teach the students, or sleep);

Your daily writing time = all the time I am thinking and writing in my mind, then I put it on paper or electronically;

Food habit = I eat all kind of food; I like to often change the food type;

Your next dream = To go to an expedition to Antarctica (that will be between 16 December 2015 - 10 January 2016).

Surapati Pramanik:

Your motivation to hard work = I do research from my own curiosity.

Florentin Smarandache:

I connect the research with our real life. For example I started the neutrosophics from my love of... soccer since I was a child! Because in soccer there are three chances: to win (<A>), to have a tie game (<neutA>), or to loose (<antiA>).

Surapati Pramanik:

Do you have any spiritual experience or feeling?

Florentin Smarandache:

I have sometimes inspiration coming… from my subconsciousness, or from the universe. When I read something, I try to generalize or to check what would have happened if it was different, or just opposite.

Surapati Pramanik:

What is your feeling of the universe?

It is bounded? …

Is it infinite?…

Florentin Smarandache:

The universe can be bounded from a point of view, and infinite from another point of view. Like the Earth, long ago, when people did not know much about… Walking continuously on the Earth, we never get to an end, but looking from the space we see that the Earth is limited. For now, the universe is so indeterminate for our minds…

*

Florentin Smarandache:

Why do you think it is important to remember how I got the idea of neutrosophics? From a moment of inspiration coming from sub-conscience…

Vic Christianto:

The reasons are both intellectual and personal:

(a) intellectual. It is very important to teach younger generation of mathematicians how was the actual discovery process, instead of talking the standard teaching that neutrosophic logic is extension of fuzzy logic. Perhaps someday people would like to improve your theory, then they should remember how you got there.

(b) personal. I was interested in the process of creative discovery. I am a fan of Edward de Bono, the father of lateral thinking. And I guess your theory has elements of lateral thinking too, because it is not logical extension or generalization of previous ideas. But only you can answer that.

Edward De Bono divides 2 thinking processes to solve problems:

(a) vertical thinking, where people use logic to analyze things. This process is linear, and uses the left side of the brain.

(b) lateral thinking, where people use creative methods which are nonlinear, and they use right side of the brain.

For example, when Archimedes used his logic to solve his problem on measuring the weight of crown, he came to dead end. Then he took a bath, and found the answer. That is lateral thinking.

Florentin Smarandache:

Lateral thinking is also a kind of outside of the mainstream thinking. A non-conventional thinking. Think differently. Be uncommon. Avoid the patterns, the routine,

and the common sense. Creative thinking. Lateral thinking will enable you to think outside of the box. "Lateral and vertical thinking are complementary" ("Lateral Thinking", by Edward de Bono, 1970).

As a poet and literary writer I used my feelings and emotions in mathematics (right part of my brain).

I started from soccer. I was a fun of my University of Craiova team, several times champion of Romania. I went to the stadium many times supporting it. I and my friends also plaid Pronosport [prognoses of the next soccer games in Romania for League A teams, a lottery with awards].

If, for example, I said the chance that my team wins is high, an enemy of my team would say that it is a high chance for my team to lose, while a third one (neutral from the point of view of supporting or rejecting my team) could say the chance of tie game would be more probable.

I observed that, considering different independent sources, the sum of these subjective probabilities can get up to $1+1+1 = 3$, which is prohibited in classical probability, where the sum of space probabilities is equal to 1.

Also, I observed that there are events that split into triads (win, loose, tie), feature that cannot be well described by fuzzy logic...

Vic Christianto:

So, you began with soccer games, which is a good story to tell. Then you can write your story that you did not get the idea for neutrosophic logic from a moment of inspiration, from gradual process of thinking & reflection.

114 Neutrosophics and Mainstream

To Dr. Surapati Pramanik:

I read the short history in your paper *Neutrosophic Linear Goal Programming* about paradigm shift...

Yes, I faced opposition, insults and humiliations from the beginning. Most of them were personal attacks, which I did not care much about. I have a proverb in my language: *The dogs bark, but the bear walks on his way anyway*.

Another thing was that $T+I+F \leq 3$, not $T+I+F \leq 1$.

So, I contradicted the classical probability, and fuzzy logic, and intuitionistic fuzzy logic.

I explained that in objective probability the sum of space objective probabilities is 1, but in subjective probability where the sources providing information on T, I, F were independent (not communicating with each other) the sum of space subjective probabilities can be extended up to 3.

Another thing was that I invented the words "neutrosophy" and its derivative "neutrosophic" (they did not exist previously of 1995 in no language). This was because "neutral / indeterminacy" was the main distinction between intuitionistic fuzzy set / logic and neutrosophic set/logic.

Neutral was also the distinction between dialectic (dynamic of opposites) and neutrosophy (dynamic of opposites and the neutral between them).

The reputed international journal *Multiple Valued Logic* (from England) in 2002 came into my help and dedicated a whole issue only to the neutrosophics.

This was the turning point, and the wheel started to move towards the neutrosophics.

Some ideas I had to publish and then republish in various extended forms in order to get attention.

This was the case of my paper on *Generalization of Intuitionistic Fuzzy Set*.

Some people working in Intuitionistic Fuzzy Set were not very happy of the Neutrosophic Set generalizing the Intuitionistic Fuzzy Set (and similarly Neutrosophic Logic generalizing Intuitionistic Fuzzy Logic). They criticized me...

In 2007 I published for the first time in the history of science the fact that the degree of membership of an element with respect to a set can be > 1 [for example, a person working overtime needs to have the degree of membership > 1 with respect to his company, while a person working only full-time has the degree of membership = 1]; and similarly the existence of degree of membership of an element with respect to a set that can be < 0 [for example, a double spy agent that leaks precious information to the enemy and produces more damage that benefit to his country should have the degree of membership with respect to his country < 0].

I expected there will be a flood of critics and insults against me, but... this passed unobserved...

Actually, the examples of overmembership and undermembership was so obvious, common sense and trivial, that no critics were directed against me.

I presented this idea (membership degrees off the interval [0, 1]) to various seminars and conferences I was invited between 1995-2016 in the world (I traveled a lot, in 52 countries) and there were many people in the audiences that agreed with me.

Thus, I got courage and I took the risk to publish a whole book only about degree of membership > 1 and degree of membership < 0.

I was very pleasantly surprised when the book was accepted by *arXiv.org* (Cornell University from New York City, which reflects the mainstream):
https://arxiv.org/ftp/arxiv/papers/1607/1607.00234.pdf
and by the big French scientific database:
https://hal.archives-ouvertes.fr/hal-01340830

The book is on neutrosophic overset (membership degree over 1), neutrosophic underset (membership degree under 0), and neutrosophic offset (some membership degrees off the interval [0, 1]).

In this book, you may see that even probability of an event can be > 1 or < 0 (neutrosophic overprobability, neutrosophic underprobability, neutrosophic offprobability). And consequently, I defined the neutrosophic overstatistics (statistics of samples or populations that have individuals whose membership degree is over 1), neutrosophic understatistics (similarly, samples or populations that have individuals with membership degree

under o), and neutrosophic offstatistics (samples or populations with individuals whose membership degree is > 1 and other individuals with membership degree < o).

I proposed REAL ideas, inspired from our world. I gave simple examples. Also, I communicated and exchanged ideas with hundreds of scientists on the globe.

Now the neutrosophics are accepted as mainstream since many websites and international journals and conferences have accepted many such papers and presentations.

I was not interested in financial profit... So I put my papers and books for free online, just for the sake of science [see my scientific site at UNM:
http://fs.gallup.unm.edu/ScienceLibrary.htm].

I live in a small rented apartment and I dedicate myself only to scientific and literary ideas [maybe you know that I also wrote poems, essays, a novel, short stories, translations, dramas, and I did art albums: see
http://fs.gallup.unm.edu/LiteraryLibrary.htm].

115 Personalia

Emenia Cera:

Ani neutrosofici, e de bine dacă ți-i urez? Cum ai vrea să-ți fie anii ce vor urma? Îți doresc mulți, frumoși si sănătoși cu împliniri pe toate domeniile! La mulți ani!

Florentin Smarandache:

La propriu e bine, la figurat nu.

Emenia Cera:

Care e diferența?

Florentin Smarandache:

La propriu ar fi: lucrări de logici / mulțimi etc. neutrosofice. La figurat ar fi: combinări de bune, rele și neutre în viață (care cam așa se întâmpla in orice viața)...

Emenia Cera:

Ambele variante sunt bine. Adică, la figurat, e normalitatea de zi cu zi...

Lotfi Zadeh
founder of Fuzzy Set

Florentin Smarandache
founder of Neutrosophic Set

My **lab**[oratory] is a virtual facility with non-controlled conditions in which I mostly perform scientific meditation and chats: *a nest of ideas* (**nidus idearum**, in Latin). In this *third book of scilogs* collected from my nest of ideas, one may find new and old questions and solutions, referring to topics on NEUTROSOPHY – email messages to research colleagues, or replies, notes about authors, articles, or books, so on. Feel free to budge in or just use the *scilogs* as open source for your own ideas!

From the Foreword